EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D9: Impulse & momentum methods

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

Acknowledgements

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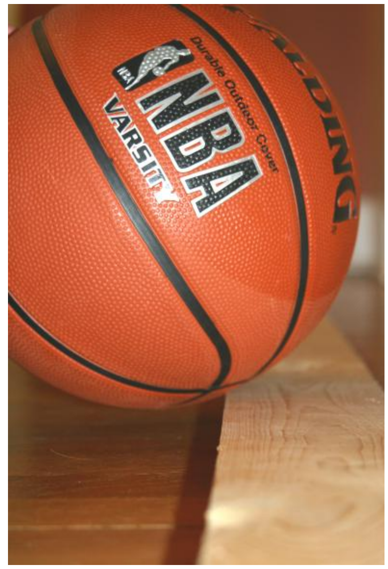
[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

PLANE MOTION OF RIGID BODIES

9. <u>Topic</u>**: Impulse and momentum methods**



Engage:

Take your bike, a basketball and a short block of wood of approximate cross-section 9cm ×2 cm.

You will need some assistance from members of the class. Place the wood block on the floor with a student standing on each end of it. On your bike freewheel very gently towards the block of wood. Repeat the experiment until your front wheel just rolls over the wood block.

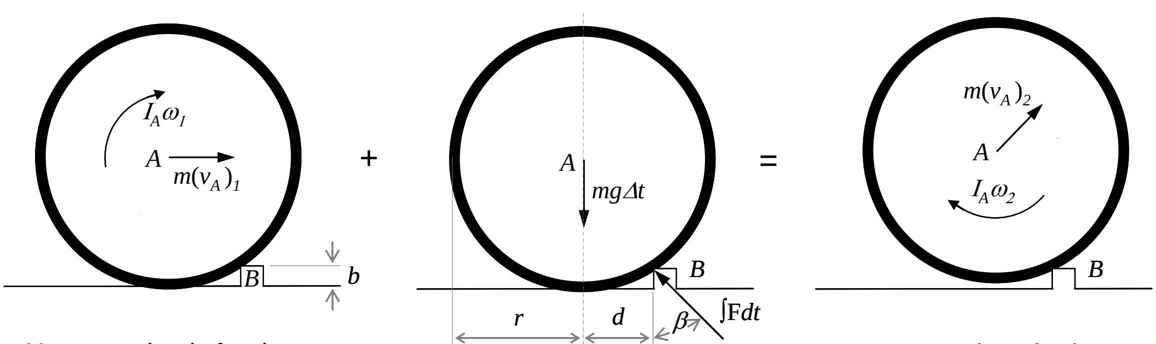
Explore:

Discuss the impulse received by your front wheel when it hits the wooden block; note that the impulse force is of unknown magnitude and direction. Highlight that angular momentum just before and just after impact must be conserved. Discuss the conservation of energy during the event and that kinetic energy at the impact must at least equal the potential energy when the wheel is on top of the wooden block in order for the wheel to roll over the block.

Actually this example is easier if you consider a basketball rolling over the block of wood, as in the photo above because then the effect of the rear wheel and the inertia of the rider does not have to be handled. However, it is less engaging and of less practical interest, so explain this to the students and repeat the demonstration for the basketball. Ask them to draw the free-body diagrams for a basketball just before and just after impact with a wooden block.

Explain:

<u>Free body diagrams:</u>



Momentum just before impact

Impulses during impact

Momentum just after impact

Let's assume that the basketball does not slip on the wooden block or rebound from it but essentially rolls over it by rotating about the corner B. Then there are two impulses or forces acting on the basketball: (a) its weight and (b) the impulse from the wooden block at B. Since the duration of impact is very small it is reasonable to assume that the weight generates a negligible impulse compared to the force at the wooden block. We do not know the magnitude or direction of the force or impulse at the wooden block.

Elaborate

Conservation of angular momentum about B from just before (1) to just after (2):

$$[\text{System Momentum}]_1 + [\text{External Impulses}]_{1\to 2} = [\text{System Momentum}]_2$$

$$m(v_A)_1(r-b)+I_A\omega_1 = m(v_A)_2 r + I_A\omega_2$$

For simplicity, consider the basketball as a hollow sphere with mass, m = 0.6kg and radius, r =0.012m:

$$I_A = \frac{2mr^2}{3} \quad \text{and} \quad \omega = \frac{v}{r},$$

so $$(v_A)_1(r-b)+\frac{2r(v_a)_1}{3} = (v_A)_2 r + \frac{2r(v_a)_2}{3}$$

Re-arranging*: $(v_A)_2 = (v_A)_1\left(1-\frac{3b}{5r}\right)$

Conservation of energy from just after impact (2) to when the basketball is on top of the wooden black (3), assuming the basketball can just mount the baton, i.e. $(v_A)_3 = 0$:

$$KE_2 + PE_2 = KE_3 + PE_3$$

$$\text{`}\left[\frac{m(v_A)_2^2}{2}+\frac{I\omega_2^2}{2}\right]+0 = 0 + mgb$$

Substituting for I and ω $(=v/r)$: $(v_A)_2^2 = \frac{6mgb}{5}$

Thus combining with conservation of angular momentum*:

$$\sqrt{\frac{6mgb}{5}} = (v_A)_1\left(1-\frac{2b}{3r}\right)$$

hence $$(v_A)_1 = \frac{\sqrt{6mgb/5}}{\left(1-\frac{2b}{3r}\right)} = \frac{\sqrt{6\times 0.6\times 9.81\times 0.02/5}}{\left(1-\frac{2\times 0.02}{3\times 0.12}\right)} = 0.422\,\text{m/s}$$

Note that as the radius of the rolling object gets larger the required velocity becomes smaller, so it easier to get a larger diameter wheel up a curb than a small one.

Evaluate

Ask students to attempt the following examples:

Example 9.1

A girl of mass 50 kg is setting off along a level sidewalk on her skateboard. If she can push off with one foot with a constant force of 150N for 0.5 seconds, calculate her speed after this push. If, almost immediately she can push again for another 0.5 seconds, then calculate her speed after this second push assuming the force applied is sinusoidal with a maximum of 150N.

Solution:

Change in Momentum = Linear Impulse

or $$mv_2 - mv_1 = \sum \int_{t_1}^{t_2} F dt$$

so, in the first push with a constant force:

$$v_2 = \frac{Ft}{m} = \frac{150 \times 0.5}{50} = 1.5 \text{ m/s } (\approx 3.36 \text{ mph})$$

and in the second push:

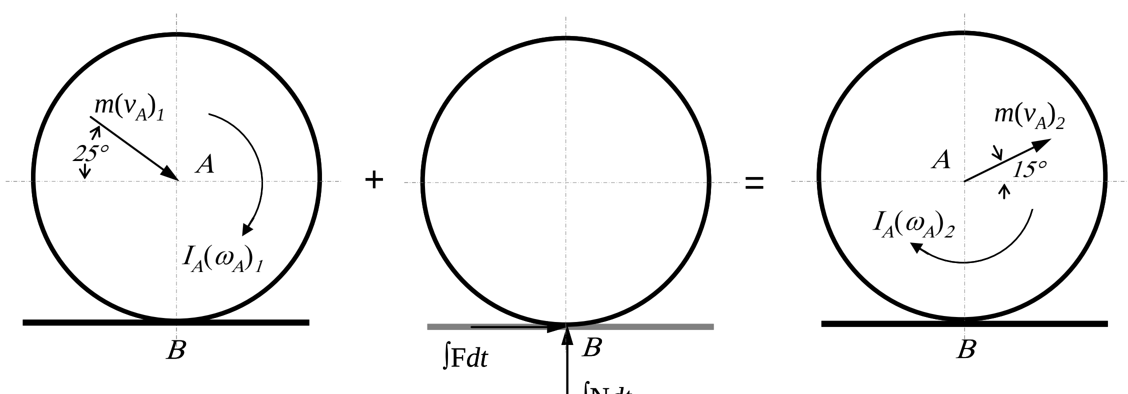
$$v_2 = \frac{\int_{0.5}^{1} 150(\sin t)\ dt + (50 \times 1.5)}{50}$$

so $$v_2 = \frac{150 \times [-\cos 1 + \cos 0.5] + 75}{50} = 2.5 \text{ m/s } (\approx 5.6\text{mph})$$

Example 9.2

Calculate the topspin (in terms of revolutions per second) required to make a tennis ball, that strikes a grass surface at 80 mph and at 25° to the horizontal, bounce at an angle of 15°, if it has a diameter of 66mm and mass of 57g.

Solution



Momentum just before impact

External impulse during impact

Momentum just after impact

Conservation of angular momentum about B:

$$m(v_A)_1 \cos 25 + I(\omega_A)_1 + 0 = m(v_A)_2 \cos 15 - I(\omega_A)_2$$

assume a perfectly elastic impact, so

$$(v_A)_1 = (v_A)_2 \text{ and } (\omega_A)_1 = (\omega_A)_2$$

and that the tennis ball is a hollow sphere, i.e. $I = \frac{2mr^2}{3}$ note that 80mph ≡ 35.8m/s

then $$\frac{4mr^2}{3}(\omega_A)_1 = mr(v_A)_1[\cos 15 - \cos 25]$$

and $$(\omega_A)_1 = \frac{3(v_A)_1[\cos 15 - \cos 25]}{4r} = \frac{3 \times 35.8[\cos 15 - \cos 25]}{4 \times 0.033} = 48.5 \text{ rads/s}$$

or about 463 rpm!