EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

T7: Gas power cycles

This is an extract from 'Real Life Examples in Thermodynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2010 (ISBN: 978-0-9842142-1-1*) and contains suggested exemplars within lesson plans for Sophomore Thermodynamics Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".*

INTRODUCTION

(from *'Real Life Examples in Thermodynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in thermodynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

This is the third in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids' (ISBN: 978-0-615-20394-2), 'Real Life Examples in Dynamics'(ISBN: 978-0-9842142-0-4).

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

POWER CYCLES

7. Topic**: Gas power cycles**

Engage:

If you have a cut-away internal combustion engine in your department, then arrange for it to be delivered to your class. In addition, or alternatively, you can show an animation of a four-stroke engine by searching in Youtube for 'Awesome Engine Animation'[4] or try: www.animatedengines.com/otto.shtml or auto.howstuffworks.com/engine1.htm

The Department of Mechanical Engineering at Michigan State University loaned one its cut-away engines to the Art Program at Okemos High School, Michigan to use as inspiration for a series of paintings. Two of the paintings, by Christine Budd (left) and Ed Emmerich (right), are reproduced above; all of the paintings are on display at MSU.

Explore:

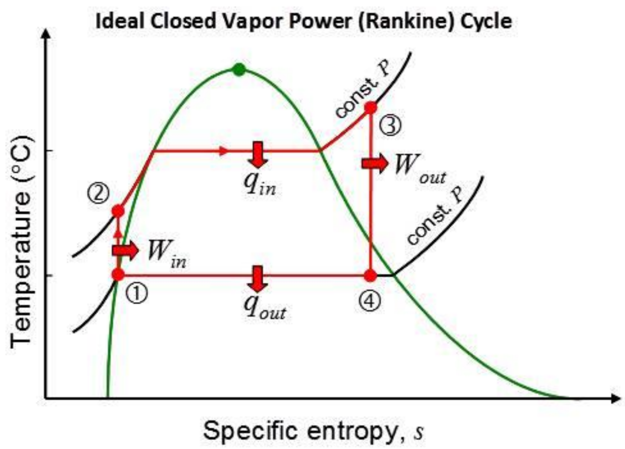
Explain the processes underway in each stroke of the engine. On the Animated Engines website you can control the speed of the cycle which allows you to explore the process in more detail.

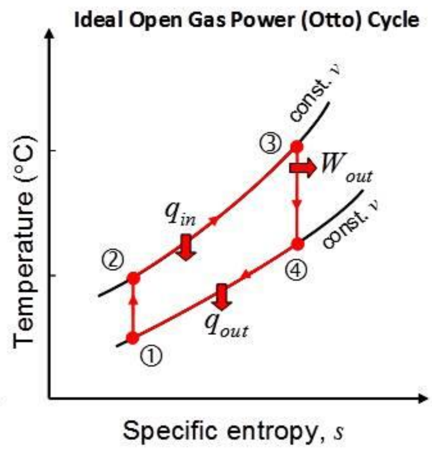
Explain:

Discuss how an internal combustion engine (or a jet engine) differs from the vapor power cycle used in a power station because (a) the working fluid always remains in the gaseous phase and; (b) it is an open cycle (as opposed to closed cycle) in which the working fluid is replaced at the end of each cycle instead of recycled.

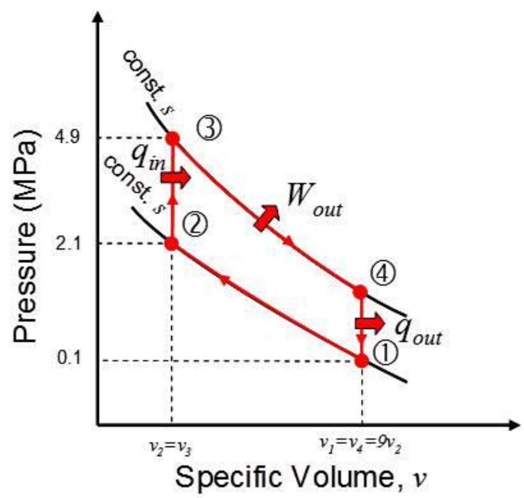
However, the ideal closed vapor power cycle and ideal open gas cycles look remarkably similar when plotted on a temperature-entropy (T-s) diagram with the closed cycle operating between two pressures and the open cycle between two specific volumes.

[4] http://www.youtube.com/watch?v=OXd1PlGur8M&NR=1&feature=fvwp





In an open power cycle, ①–② is an isentropic compression of the fuel-air mixture by the piston, ②–③ is a constant volume heat transfer (addition) during combustion of the fuel, ③–④ is an isentropic expansion as the piston is pushed down the cylinder and ④–① is a constant-volume heat transfer (rejection) as the exhaust gases are pushed out of the cylinder.



Elaborate:

An Otto engine takes in air from its surroundings so that:

State 1: $P_1 = 101\,\text{kPa}$ and $T_1 = 21°\text{C} \equiv 294\,\text{K}$ these values can be used in the thermodynamic tables[5] to obtain the internal energy, $u_1 = 209.774\,\text{kJ/kg}$ and the relative specific volume, $v_{r1} = 653.54$ of air as an ideal gas.

Process ①–②: (*an isentropic compression*)

If an Otto engine has a compression ratio, $r = 9$,

then $$r = \frac{v_1}{v_2} = \frac{v_{r1}}{v_{r2}} = 9$$ and

$$v_{r2} = \frac{v_{r1}}{9} = \frac{653.54}{9} = 72.62$$

hence from the thermodynamic tables[6]: $T_2 = 689.9\,\text{K}$ and $u_2 = 504.015\,\text{kJ/kg}$.

Now, $$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \text{ so } P_2 = P_1 \frac{V_1}{V_2}\left(\frac{T_2}{T_1}\right) = P_1 r \left(\frac{T_2}{T_1}\right) = 101 \times 9 \times \left(\frac{689.9}{294}\right) = 2133\,\text{kPa}$$

Process ②–③: (*constant volume heat transfer*)

[5] http://v5.sdsu.edu/testhome/Test/solve/basics/tables/tables.html
[6] http://v5.sdsu.edu/testhome/Test/solve/basics/tables/tables.html

If 17g of gasoline (with a Gross Calorific Value, GCV = 47,300 kJ/kg) is injected into the cylinder and completely combusts, then the heat addition will be

$$q_{in} = m \times GCV = 0.017 \times 47300 = 804.1\,\text{kJ}$$

And applying the first law of thermodynamics for the isentropic compression by equating the heat input to the change in internal energy, noting that we can consider internal energy rather than enthalpy for the constant volume process,

$$q_{in} = u_3 - u_2 \text{ so } u_3 = q_{in} + u_2 = 804.1 + 504.015 = 1308.115\,\text{kJ/kg}$$

so from the thermodynamic tables: $T_3 = 1610.5\,\text{K}$ and $v_{r3} = 5.683$.

Now, $\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2}$ so $P_3 = P_2\left(\frac{T_3}{T_2}\right)\left(\frac{v_2}{v_3}\right) = 2133 \times \left(\frac{1610.5}{689.9}\right) \times 1 = 4979.26\,\text{kPa}$

<u>Process ③–④</u>: (*isentropic expansion*)

Again $r = \frac{v_4}{v_3} = \frac{v_{r4}}{v_{r3}} = 9$ and $v_{r4} = r v_{r3} = 9 \times 5.583 = 50.247$

so from the thermodynamic tables: $T_4 = 787.83\,\text{K}$ and $u_4 = 582.45\,\text{kJ/kg}$

<u>Process ④–①</u>: (*constant-volume heat transfer (rejection)*)

Again, using the first law of thermodynamics to equate heat transfer to the change in internal energy

$$q_{out} = u_4 - u_1 = 582.45 - 209.774 = 372.676\,\text{kJ/kg}$$

Finally, $w_{net} = q_{in} - q_{out} = 804.1 - 372.676 = 431.424\,\text{kJ/kg}$

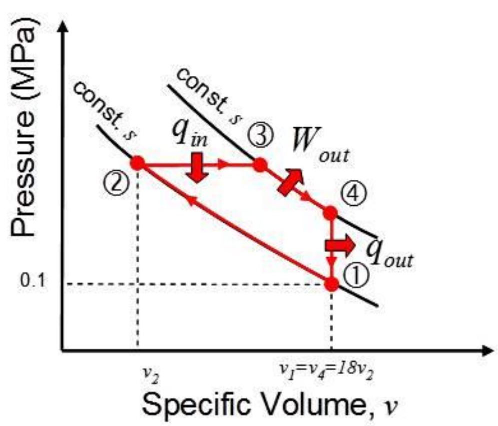
Giving a thermal efficiency, $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{431.424}{804.1} = 0.5365$ or 53.6%

Evaluate:

Invite students to attempt the following examples:

<u>Example 7.1</u>

Given that an ideal diesel engine has a cycle as shown, i.e. the combustion process occurs at constant pressure rather constant volume as in a spark ignition engine:



Determine the cutoff ratio, $r_c = V_3/V_2$ i.e. the ratio of the volumes after and before combustion at which the ideal diesel engine must operate if it is to achieve the same thermal efficiency with the same fuel input as the spark ignition engine represented by the ideal Otto cycle above but with twice the compression ratio.

<u>Solution</u>

By definition: $\eta_{th} = \dfrac{w_{net}}{q_{in}}$ and $\eta_{th} = 0.5365$, $q_{in} = 804.1\,\text{kJ/kg}$

so $\quad w_{net} = \eta_{th} q_{in} = 0.5365 \times 804.1 = 431.424\,\text{kJ/kg}$

And $\quad w_{net} = q_{in} - q_{out}$ so $q_{out} = q_{in} - w_{net} = 804.1 - 431.424 = 372.676\,\text{kJ/kg}$

Also, by applying the first law of thermodynamics

$$q_{out} = u_4 - u_1 = 372.676\,\text{kJ/kg} \text{ and } q_{in} = h_3 - h_2 = 804.1\,\text{kJ/kg}$$

(note: q_{in} does not happen at constant volume so it is necessary to use enthalpy, i.e. $h = u + pv$)

<u>State 1</u>: remains unchanged, i.e. $P_1 = 101\,\text{kPa}$ and $T_1 = 21°\text{C} \equiv 294\text{K}$ and from thermodynamic tables[15]: $u_1 = 209.774\,\text{kJ/kg}$ and the relative specific volume, $v_{r1} = 653.54$

So for a compression ratio of 18, $r = \dfrac{v_1}{v_2} = \dfrac{v_{r1}}{v_{r2}} = 18$ and $v_{r2} = \dfrac{v_{r1}}{18} = \dfrac{653.54}{18} = 36.31$

<u>State 2:</u> given v_{r2} = 36.31 then from the thermodynamic tables: $T_2 = 882.6\,\text{K}$ and $h_2 = 894.957$ kJ/kg.

now, $\quad \dfrac{P_2 v_2}{T_2} = \dfrac{P_1 v_1}{T_1}$ so $P_2 = P_1 r \left(\dfrac{T_2}{T_1}\right) = 101 \times 18 \times \left(\dfrac{882.6}{294}\right) = 5457.71\,\text{kPa}$

From $\quad q_{in} = h_3 - h_2 = 804.1\,\text{kJ/kg}$,

$$h_3 = q_{in} + h_2 = 804.1 + 894.957 = 1699.057\,\text{kJ/kg}$$

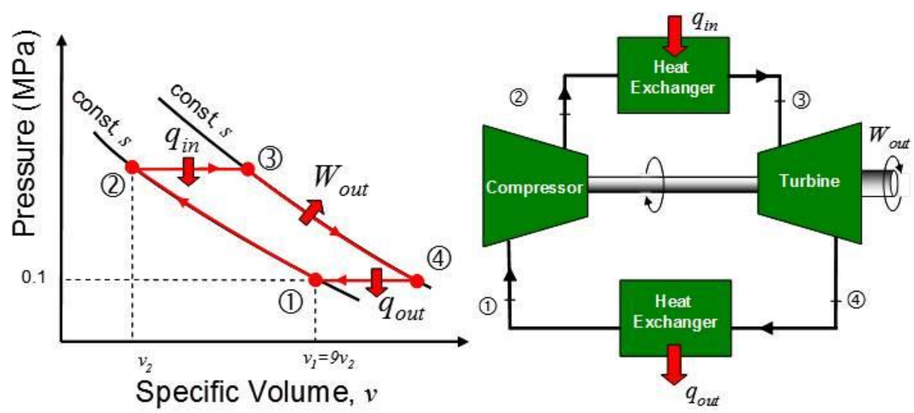
And from the thermodynamic tables: $T_3 = 1548.03\,\text{K}$

<u>Process ②–③</u> occurs at constant pressure, so $\dfrac{P_2 v_2}{T_2} = \dfrac{P_3 v_3}{T_3}$ becomes $T_3 = \dfrac{v_3}{v_2} T_2$

and, by definition, the cutoff ratio, $r_c = \dfrac{v_3}{v_2} = \dfrac{T_3}{T_2} = \dfrac{1548.03}{882.6} = 1.754$

<u>Example 7.2</u>

The ideal cycle for a gas turbine, known as the Brayton cycle, has a constant pressure process of heat rejection as shown below. If it were operated over the same temperature range as the diesel engine in the previous example but with the pressure ratio of the Otto cycle above, what would be the thermal efficiency?



Solution:

State 1: remains unchanged, i.e. $P_1 = 101\text{kPa}$ and $T_1 = 21°\text{C} \equiv 294\text{K}$ and from thermodynamic tables[14]: $h_1 = 294.17$ kJ/kg and the relative pressure, $P_{r1} = 1.2917$

Process ①–②: (*compression of gas*)

Relative pressure at the compressor outlet, $P_{r2} = \frac{P_2}{P_1} P_{r1} = 9 \times 1.2917 = 11.6245$

Hence from thermodynamics tables: $T_2 = 546.91\text{K}$ and $h_2 = 552.22\,\text{kJ/kg}$

Process ③–④: (*expansion of gas*)

The gas is heated to $T_3 = 1548.03\text{K}$ in the heat exchanger

so from thermodynamics tables: $h_3 = 1699.07$ kJ/kg and $P_{r3} = 687.88$

and $P_{r4} = \frac{P_4}{P_3} P_{r3} = \left(\frac{1}{9}\right) \times 687.88 = 76.43$

so from thermodynamics tables: $T_4 = 903.38\,\text{K}$ and $h_4 = 936.72\,\text{kJ/kg}$.

Hence, using the first law of thermodynamics

$$w_{in} = h_2 - h_1 = 522.22 - 294.17 = 228.05\,\text{kJ/kg}$$

and $w_{out} = h_3 - h_4 = 1699.07 - 936.72 = 762.37$ kJ/kg

thus, $w_{net} = w_{out} - w_{in} = 762.37 - 228.05 = 534.32$ kJ/kg

And $q_{in} = h_3 - h_2 = 1699.07 - 522.22 = 1176.85\,\text{kJ/kg}$

Hence by definition, $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{534.32}{1176.85} = 0.454$ or 45.5%