EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

T10: Psychrometric applications

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This is an extract from 'Real Life Examples in Thermodynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2010 (ISBN: 978-0-9842142-1-1) and contains suggested exemplars within lesson plans for Sophomore Thermodynamics Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: “Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change”.

INTRODUCTION

(from 'Real Life Examples in Thermodynamics: Lesson plans and solutions')

These notes are designed to enhance the teaching of a sophomore level course in thermodynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study\(^1\) in the 1980s from work by Atkin & Karplus\(^2\) in 1962. Today this approach is considered to form part of the constructivist learning theory\(^3\).

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.


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THERMODYNAMIC APPLICATIONS

10. Topic: Psychrometric applications

Engage:
Take a bottle of water into class. Make sure that the bottle is really cold so that it will ’sweat’ in class. Place it on a napkin then show it to the students so they can see the ring mark.

If you wear glasses, take them off and breathe on the lenses then invite all of the students in the class wearing glasses to do the same and show the results to those who are not wearing glasses.

Also borrow a wet and dry bulb thermometer from the lab., preferably in the form of a sling psychrometer.

Explore:
Ask students working in pairs to discuss the connection between the sweating water bottle, the steamed up glasses, and the early morning dew on the grass. Invite a couple of pairs to provide their thoughts on the issue.

Explain:
The air in the room can be considered as a mixture of air and water vapor. The mixture can be considered to be made up of ideal gases so the pressure, $P$, is the sum of the partial pressures of the air, $P_a$, and water vapor, $P_v$.

In the room the partial pressures are constant and the water is completely vapor, i.e. it can be represented by $\circ$ in the T-s graph. The air adjacent to the bottle is cooled at constant vapor pressure until it reaches the saturation line, i.e. $T_{sat}$ at $\circ$, which is also known as the dew-point and liquid water is formed.

Your breathe is at about 35°C and 95% humidity\(^4\) i.e. a mixture of water vapor and in the ratio of masses of 0.95 to 0.05. When this mixture meets the cold surface of the glasses it is cooled to below the dew-point and the water vapor condenses.

At night, thin, exposed objects radiate their heat at a faster rate than it can be replaced and so they cool relative to the environment. The air adjacent to them also cools and if it cools below its dew-point then droplets of liquid water will form. If this process occurs is followed by sub-zero temperatures at night, then the dew freezes to form a frost.

\(^4\)http://www.sciencebits.com/exhalecondense
Elaborate:
Spin the sling psychrometer and obtain wet and dry bulb temperatures for the room.

When the wet bulb thermometer is spun in the unsaturated air some water evaporates from the wet bulb. This process can be analyzed by considering the motion of the air relative to the bulb; the air goes from state ① before passing across the wet bulb to state ② after passing across the wet bulb.

By definition the specific humidity is
\[ \omega = \frac{m_v}{m_a} = \frac{P_v/RT}{P_a/RT} = \frac{P_v}{P_a} \]

Substituting for \( P = P_a + P_v \) and given that \( R_v/R_a = 0.622 \) gives
\[ \omega = \frac{0.622P_v}{P - P_v} \]

So for a wet bulb temperature of \( 15^\circ C \) (= \( T_2 \)) the vapor is saturated and the saturation pressure can be obtained in the thermodynamic tables as \( 1.7051 \) kPa and
\[ \omega_2 = \frac{0.622P_v}{P - P_v} = \frac{0.622 \times 1.7051 \times 10^3}{(101 - 1.7051) \times 10^3} = 0.01068 \]

Mass flow rates:
The mass flow rate of the dry air remains constant, i.e., \( \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a \)
The mass flow rate of the water vapor increases by the amount which evaporates from the wet bulb, \( \dot{m}_v \), i.e. \( \dot{m}_{v_1} + \dot{m}_e = \dot{m}_{v_2} \)

So \( \dot{m}_e = \dot{m}_{v_2} - \dot{m}_{v_1} \) and substituting \( \omega = \frac{m_v}{m_a} \) yields
\[ \dot{m}_e = \dot{m}_v (\omega_2 - \omega_1) \]

1st law of thermodynamics applied to the process with no heat transfer and no work is:
\[ \dot{E}_m = \dot{E}_{out} \text{ or } \dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{v_1} + \dot{m}_e h_e = \dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2} \]

Now substituting for \( \dot{m}_e \), dividing by \( \dot{m}_a \) and rearranging yields for specific humidity prior to passing over the wet bulb
\[ \omega_1 = \frac{h_{a_2} - h_{a_1} + \omega_2 (h_{v_2} - h_e)}{h_{v_1} - h_e} \]

If the air-vapor mixture is an ideal gas then \( h = f_n(T) \) and
\[ h_{a_2} - h_{a_1} = C_p (T_2 - T_1) \]

Note that \( h_{v_1} \) and \( h_{v_2} \) are the enthalpy of saturated vapor, i.e. \( h_s \) at \( T_1 \) and \( T_2 \) respectively, and \( h_e \) is the enthalpy of saturated liquid at \( T_2 \), i.e. \( h_f \), so \( (h_{v_2} - h_e) = h_{fs} \), then
\[ \omega_1 = \frac{C_p (T_2 - T_1) + \omega_2 h_{fs}}{h_{f_1} - h_{f_2}} \]
Again from the thermodynamic tables at $T_2 = 15^\circ\text{C}$, $h_{fs_2} = 2465.91 \text{ kJ/kg}$ and $h_{f_2} = 62.99 \text{ kJ/kg}$; and for a dry bulb temperature, $T_1 = 20^\circ\text{C}$ then $h_{g_1} = 2538.1 \text{ kJ/kg}$ so specific humidity in the room is

$$\omega_1 = \frac{1.005(15 - 20) + 0.01068 \times 2465.91}{2538.1 - 62.99} = 0.00861$$

Now, also $\omega = \frac{0.622P_v}{P - P_v}$ so $P_v = \frac{\omega P}{0.622 + \omega}$ and $P_{v_i} = \frac{0.00861 \times 101}{0.622 + 0.00861} = 1.379 \text{ kPa}$

Hence the dew-point at this pressure from the thermodynamic tables is $(T_{sat})_{p=1.379 kPa} = 11.57^\circ\text{C}$

The relative humidity, $\phi$ is defined as the ratio of the mass of water vapor in air to the maximum mass of water vapor that could be held at the same temperature, $m_g$, i.e.,

$$\phi = \frac{m_v}{m_g} = \frac{P_v V}{P_g V} = \frac{P_v}{P_g}$$

where $P_g = P_{sat}$ at the considered temperature. So for the relative humidity of the room

$$\phi_i = \frac{P_{v_i}}{(P_{sat})_{p=1.379 kPa}} = \frac{1.379}{2.339} = 0.590 \text{ or } 59\%$$

The same calculation can be performed on a psychrometric chart\(^5\) as shown above.

**Evaluate:**

Invite the students to attempt the following examples:

*Example 10.1*

In the morning when you get into your car which has been standing out overnight in temperatures at about freezing, the windshield starts to mist up from your breathe. Determine the temperature to which the screen must be raised by the screen heater to prevent the misting.

**Solution:**

You exhale at about $34^\circ\text{C}$ and $95\%$ relative humidity, and from the thermodynamics tables\(^{15}\) $P_{sat} = 5.3516 \text{ kPa}$ at $T = 34^\circ\text{C}$

So $P_v = \phi P_{sat} = 0.95 \times 5.3516 = 5.08 \text{ kPa}$

And from the thermodynamics tables, $T_{sat} = 33.1^\circ\text{C}$ for $5.08 \text{ kPa}$ which is the dew-point for the air breathed out so the windshield must be warmer than $33.1^\circ\text{C}$ to prevent your breathe condensing on it. However your breathe will mix with the air in the vehicle and after a few

\(^{5}\text{ e.g. http://v5.sdsu.edu/testhome/Test/solve/basics/tables/tablesMA/psychro.html}\)
minutes the average temperature in the car might have risen from about 0°C to 16°C depending on the size of the car and the number of passengers. At the same time the relative humidity might have halved to around 50%. A psychometric chart can be used to estimate the dew-point in these conditions, i.e. \( T_{dp} = 5^\circ C \) so the surface of the wind shield would need to be at least 5°C to prevent misting.

**Example 10.2**

After leaving the condenser of a power station, water enters the top of a cooling tower as a spray and falls over baffles to the bottom where it is recirculated to the power plant. At the same time air enters the tower at the bottom and is sucked up by a fan and billows out of the top generating huge white clouds in certain weather conditions.

The water enters the tower at 35°C and 120 kg/s and is cooled to 25°C by the air that enters at 20°C, 50% relative humidity and atmospheric pressure. If the air leaves saturated at 30°C, calculate required the flow rate of the air.

**Solution:**

From the thermodynamic tables for saturated water\(^6\)

\[
h_{T=35} = h_3 = 146.68 \text{ kJ/kg}, \quad h_{T=25} = h_4 = 104.89 \text{ kJ/kg}
\]

From a psychrometric chart\(^2\) at \( T_1 = 20^\circ C \) and \( \phi_1 = 50 \%

\[
h_1 = 39.5 \text{ kJ/kg dry air},
\]

\[
\omega_1 = 0.00725 \text{ kg H}_2\text{O/kg dry air},
\]

\[
v_1 = 0.84 \text{ m}^3/\text{kg dry air}
\]

And using \( T_2 = 30^\circ C \) and \( \phi_2 = 100 \%

\[
h_2 = 100.0 \text{ kJ/kg}
\]

\[
\omega_2 = 0.0273 \text{ kg H}_2\text{O/kg dry air}
\]

Mass balance for the dry air \( \dot{m}_a_1 = \dot{m}_a_2 = \dot{m}_a \)

Mass balance for the water \( \dot{m}_3 + \dot{m}_a \omega_1 = \dot{m}_4 + \dot{m}_a \omega_2 \) or \( \dot{m}_3 - \dot{m}_4 = \dot{m}_a (\omega_2 - \omega_1) \)

Energy balance from the first law of thermodynamics

\[
\sum_{in} \dot{m}h = \sum_{out} \dot{m}h \quad \text{so} \quad \dot{m}_3 h_3 + \dot{m}_a h_1 = \dot{m}_4 h_4 + \dot{m}_a h_2
\]

So combining the above two

\[
\dot{m}_a = \frac{\dot{m}_3 (h_3 - h_1)}{(h_2 - h_1) - h_4 (\omega_2 - \omega_1)} = \frac{120(146.68 - 104.89)}{(100 - 39.5)\times104.89 	imes(0.0273 - 0.00725)} = 85.87 \text{ kg/s}
\]

\(^6\) [http://v5.sdsu.edu/testhome/Test/solve/basics/tables/tables.html](http://v5.sdsu.edu/testhome/Test/solve/basics/tables/tables.html)