EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

S6: Conservation of energy

This is an extract from 'Real Life Examples in Mechanics of Solids: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Mechanics of Solids: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore course in mechanics of solids, increase the accessibility of the principles and raise the appeal of the subject to students from a diverse background[1]. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. These are not original and were developed by the Biological Sciences Curriculum Study[2] in the 1980s from work by Atkin and Karplus[3] in 1962. Today they are considered to form part of the constructivist learning theory and a number of websites provide easy to follow explanations of them[4].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

Acknowledgements

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[1] Patterson, E.A., Campbell, P.B., Busch-Vishniac, I., Guillaume, D.W., 2011, The effect of context on student engagement in engineering, *European J. Engng Education*, 36(3):211-224.
[2] http://www.bscs.org/library/BSCS_5E_Instructional_Approach_July_06.pdf
[3] Atkin, J. M. and Karplus, R. (1962). Discovery of invention? *Science Teacher* 29(5): 45.
[4] e.g. http://www.science.org.au/primaryconnections/constructivist.htm

STRAIN ENERGY

6. Principle: **Conservation of Energy / Energy Methods**

Engage

Bring a slingshot and a handful of rubber balls into class, pull the elastic band back and release a few rubber balls into the class.

Explore

Ask students to work in pairs and to identify the conservation of energy during the loading, firing and trajectory of the balls. Invite some pairs to talk through to the class their understanding of the energy conversions. Tell them about elastic strain energy stored in the elastic band. Discuss how strain energy is stored in the material and is available for instantaneous release. Ask them in their pairs to reconsider conservation of energy during loading, firing and flight of projectile.

Explain

Strain energy is defined as the energy stored in a material when work has been done on the material, assuming the material remains elastic and no permanent deformation occurs. Equate strain energy to work done and derive an expression relating strain energy, U to deflection, δ and applied force, F i.e. $U = \frac{1}{2} F\delta$. Consider the stress-strain curve for rubber and hence estimate the strain energy stored per unit volume, u upto the elastic limit ($u \approx 3.6\,\text{MJm}^{-3}$).

Elaborate

Estimate release velocity of projectile, i.e.

For an elastic band of length, 150mm and cross-section 16mm^2 the volume, V is 2400mm^3, so the energy stored when pulled to yield:

$$U = uV = (3.6\times10^{6})(24\times10^{-9}) = 8.6\,\text{J}.$$

$$\text{Kinetic energy supplied} = \frac{1}{2}Mv^2,$$

so equating strain energy stored with kinetic energy supplied to a yellow dot squash ball:

$$v = \sqrt{\frac{2U}{M}} = \sqrt{\frac{2\times8.6}{24\times10^{-3}}} = 27\,\text{ms}^{-1}$$

Evaluate

Ask students to attempt the following examples:

Example 6.1

Consider a bungee jumper with a mass of 50kg leaping from a bridge who breaks their fall with a long elastic cord having an axial rigidity, *EA*=2.1kN. If the bridge parapet is 60m above the water, and if it is desired to maintain a clearance of 10m between the jumper and the water, calculate the length of cord that should be used.

Solution

Potential energy lost by jumper = work done on cord or strain energy gained

So, $$mgh = \frac{F\delta}{2} \qquad \text{(i)}$$

where m is the mass of the jumper, g is acceleration due to gravity, h is the distance fallen by jumper, F is the maximum force in the cord and δ is the maximum extension of the cord. Now, by definition:

$$\delta = \varepsilon L = \frac{\sigma L}{E} = \frac{FL}{AE}, \text{ so } F = \frac{\delta AE}{L} \qquad \text{(ii)}$$

Substituting in (i) gives:

$$mgh = \frac{EA\delta^2}{2L} \qquad \text{(iii)}$$

Also know that $h = L + \delta$, so substituting in (iii) leads to:

$$mgh = \frac{EA(h-L)^2}{2L} = \frac{EA(h^2 - 2hL + L^2)}{2L}$$

So, $$2mghL = EAh^2 - 2EAhL + EAL^2$$

Thus, $$EAL^2 - 2(EA + mg)hL + EAh^2 = 0$$

Solution is $$L = \frac{2h(EA+mg) \pm \sqrt{4h^2(EA+mg)^2 - 4(EAh)^2}}{2EA}$$

$$L = \frac{(2\times 50)(2.1\times 10^3 + 50\times 9.81) \pm \sqrt{(4\times 50^2)(2.1\times 10^3 + 50\times 9.81)^2 - 4\times(2.1\times 10^3 \times 50)^2}}{2\times(2.1\times 10^3)}$$

$$L = \frac{259050 \pm 151680}{4200} = 25.6\,\text{m}$$

Required length of cord is 25.6m.

Example 6.2

Ask students to look for two other examples in their everyday life and explain how the above principles apply to each example