EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

S3: Pressure vessel stresses

This is an extract from 'Real Life Examples in Mechanics of Solids: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Mechanics of Solids: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore course in mechanics of solids, increase the accessibility of the principles and raise the appeal of the subject to students from a diverse background[1]. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. These are not original and were developed by the Biological Sciences Curriculum Study[2] in the 1980s from work by Atkin and Karplus[3] in 1962. Today they are considered to form part of the constructivist learning theory and a number of websites provide easy to follow explanations of them[4].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

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[1] Patterson, E.A., Campbell, P.B., Busch-Vishniac, I., Guillaume, D.W., 2011, The effect of context on student engagement in engineering, *European J. Engng Education*, 36(3):211-224.
[2] http://www.bscs.org/library/BSCS_5E_Instructional_Approach_July_06.pdf
[3] Atkin, J. M. and Karplus, R. (1962). Discovery of invention? *Science Teacher* 29(5): 45.
[4] e.g. http://www.science.org.au/primaryconnections/constructivist.htm

ELEMENTARY STRESS SYSTEMS

3. Principle: **Pressure vessel stresses**

Engage:

Take the front wheel from your bike into class with a large bike pump. Its good to have a flat tire, but no puncture. Inflate the tire in front of the class. Ask them to what pressure you should inflate the tire. Answer is written on the wall of the tire.

Explore:

Compare with the pressure for your car tires. Ask students in pairs to discuss:

(a) why it is difficult to pump up car tires with a bike pump (displaced volume of pump);

(b) the force you need to exert to pump against maximum pressure ($F = p_{max} A$ where A is the cross-section area of piston in pump**)** and

(c) the work done on each stroke ($W = Fd$ where d is the length of the stroke, i.e. distance moved)

Then invite different pairs to give an explanation to each problem.

Explain:

The tire and inner tube is a complex stress system. First, due to the toroidal geometry, and second, because the tire constrains the free expansion of the inner tube, reducing tensile stresses in tube. However, the pump can be considered as a simple cylindrical pressure vessel and wall stresses can be evaluated.

Elaborate:

Work through the example below:

Assume pumping against maximum pressure allowed in the tire as a worse case scenario, let $p_{max} = 6$ bar with a pump of external diameter 40mm and wall thickness 1.25mm. Evaluate the circumferential and longitudinal stress in the wall of pump. Compare these stresses to the stress in the rod connecting the piston to the handle if it is made of 10mm diameter tube with a wall thickness of 2mm.

Solution

Circumferential stress, $\sigma_1 = \frac{pr}{t} = \frac{\left(6\times 10^5\right)\times 0.02}{1.25\times 10^{-3}} = 9.6\times 10^6 \text{ N/m}^2$

where *r* is the radius of the cylinder and *t* is the wall thickness

Longitudinal stress, $\sigma_1 = \frac{pr}{2t} = \frac{\left(6\times 10^5\right)\times 0.02}{(2)\left(1.25\times 10^{-3}\right)} = 4.8\times 10^6 \text{ N/m}^2$

Rod stress, $\sigma = \frac{F}{A} = \frac{pA_{piston}}{A_{rod}} = \frac{\left(6\times 10^5\right)\left(38.5\times 10^{-3}\right)^2}{\left(10^2 - 6^2\right)\times 10^{-6}} = 13.9\times 10^6 \text{ N/m}^2$

Note that the compressive stress in the rod is higher, so compare with yield stresses for material of your pump. Discuss the probability of failure, actually the rod is more likely to fail in buckling.

Evaluate:

Ask students to design an emergency foot-pump for pumping up car tires. Ask them to design it in aluminum or plastic so that it is lightweight and easy to handle.