EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

S12: Combined bending & torsion

This is an extract from 'Real Life Examples in Mechanics of Solids: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Mechanics of Solids: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore course in mechanics of solids, increase the accessibility of the principles and raise the appeal of the subject to students from a diverse background[1]. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. These are not original and were developed by the Biological Sciences Curriculum Study[2] in the 1980s from work by Atkin and Karplus[3] in 1962. Today they are considered to form part of the constructivist learning theory and a number of websites provide easy to follow explanations of them[4].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

Acknowledgements

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[1] Patterson, E.A., Campbell, P.B., Busch-Vishniac, I., Guillaume, D.W., 2011, The effect of context on student engagement in engineering, *European J. Engng Education*, 36(3):211-224.
[2] http://www.bscs.org/library/BSCS_5E_Instructional_Approach_July_06.pdf
[3] Atkin, J. M. and Karplus, R. (1962). Discovery of invention? *Science Teacher* 29(5): 45.
[4] e.g. http://www.science.org.au/primaryconnections/constructivist.htm

TWO-DIMENSIONAL STRESS SYSTEMS

12. Principle: **Combined bending and torsion**

Engage

Take a few old or cheap wind-up alarm clocks into class. Set one to go off at the start of the lecture! Dismantle them to various stages and pass around.

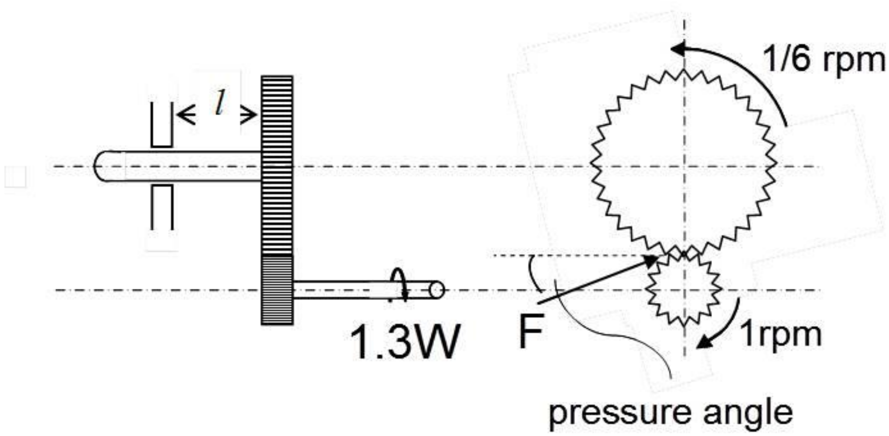
Explore

Discuss the forces, moments and torques acting inside the mechanism. Ask them in pairs to figure out how the clock mechanism works.

See http://home.howstuffworks.com/inside-clock.htm

Explain

Explain that gears are often overhung and hence their shafts are subject to both torsion and bending.



Elaborate

Work through the example below:

The clock drives the minute hand through a spur reduction gear. The 4mm diameter drive wheel transmits 2mW at 1 r.p.m. and has a pressure angle of 20º. The driven wheel has a diameter of 24mm and is overhung 6mm from its bearing.

$$\text{Power, } P = \frac{T\theta}{t} = 2\pi NT$$

where T is torque, θ is angle turned, t is time taken, and N is rotational speed (revs.s^{-1}).

For the driven (smaller) gear:

Torque, $T = \frac{P}{2\pi N} = \frac{0.002}{2\pi(1/60)} = 0.019\,\text{Nm}$

Circumferential force, $F_{circ} = \frac{T}{r} = \frac{0.0019}{\left(4\times10^{-3}\right)/2} = 9.55\,\text{N}$

And from the geometry of the pressure angle:

$$F_{circ} = F\cos 20\text{, so } F = \frac{F_{circ}}{\cos 20} = 10\,\text{N}$$

For the larger gear:

Torque in shaft, $T = Fr\sin 80 = 11\times 0.012\sin 80 = 0.12\,\text{Nm}$

Moment of shaft, $M = Fl = 11\times 0.006 = 0.06\,\text{Nm}$

And, shear stress due to torque, $\tau_{xy} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3}$

stress due to bending moment, $\sigma_x = \frac{My}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3}$

Using a Mohr's circle of stress:

$$\sigma_{\max} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

Therefore $d = \sqrt[3]{\frac{16}{\pi\sigma_{\max}}\left(M + \sqrt{M^2 + T^2}\right)}$ or $d = \sqrt[3]{\frac{16}{\pi\tau_{\max}}\sqrt{M^2 + T^2}}$ whichever is greater.

If the gears are made from polythene with a tensile strength of 30MPa, then it would be reasonable that the maximum tensile stress should not exceed 15MPa and the maximum shear should not exceed 7MPa, hence the diameter of the driven gear shaft must be at least 4.6 mm based on shear stress and 4.0mm based on tensile stress, so the minimum shaft diameter is 4.6mm.

Evaluate

Ask students to attempt the following examples:

<u>*Example 12.1*</u>

A machine is driven by an 8kW motor through a spur reduction gear, the motor runs at 1450 r.p.m. and its driving pinion has 21 teeth. The driven wheel on the machine has 100 teeth and is overhung 120mm from a main bearing. The teeth of the gear are of 3mm module and the pressure angle is 20º. Find a suitable diameter for the shaft at the main bearing of the machine if the tensile stress is limited to 80MPa and the shear stress to 50MPa when the motor is developing its rated power.

Solution:

As above, for the small gear

Torque, $T = \frac{P}{2\pi N} = \frac{8\times10^3}{2\pi(1450/60)} = 52.69\,\text{Nm}$

Now, Module = pitch diameter/number of teeth

So Pitch diameter = 21 × 3 = 63mm

Circumferential force, $F_{circ} = \frac{T}{r} = \frac{52.69}{(63\times10^{-3})/2} = 1.672\times10^3\,\text{N}$

And from the geometry of the pressure angle:

$$F_{circ} = F\cos 20\text{, so } F = \frac{F_{circ}}{\cos 20} = 1.779\times10^3\,\text{N}$$

For the larger gear:

Pitch diameter = number of teeth × module = 100 × 3 = 300mm

Torque in shaft, $T = Fr\sin 80 = 1779\times0.150\sin 80 = 262.8\,\text{Nm}$

Moment of shaft, $M = Fl = 1779\times0.120 = 213.5\,\text{Nm}$

Thus,

$$d = \sqrt[3]{\frac{16}{\pi\sigma_{\max}}\left(M+\sqrt{M^2+T^2}\right)} = \sqrt[3]{\frac{16}{\pi(80\times10^6)}\left(213.5+\sqrt{213.5^2+262.8^2}\right)} = 32.75\,\text{mm}$$

And

$$d = \sqrt[3]{\frac{16}{\pi\tau_{\max}}\sqrt{M^2+T^2}} = \sqrt[3]{\frac{16}{\pi(50\times10^6)}\sqrt{213.5^2+262.8^2}} = 32.55\,\text{mm}$$

So the diameter must be at least 32.75mm.

Example 12.2

Ask students to look for two other examples in their everyday life and explain how the above principles apply to each example.