EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

S10: Thermal loading

This is an extract from 'Real Life Examples in Mechanics of Solids: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Mechanics of Solids: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore course in mechanics of solids, increase the accessibility of the principles and raise the appeal of the subject to students from a diverse background[1]. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. These are not original and were developed by the Biological Sciences Curriculum Study[2] in the 1980s from work by Atkin and Karplus[3] in 1962. Today they are considered to form part of the constructivist learning theory and a number of websites provide easy to follow explanations of them[4].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

Acknowledgements

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[1] Patterson, E.A., Campbell, P.B., Busch-Vishniac, I., Guillaume, D.W., 2011, The effect of context on student engagement in engineering, *European J. Engng Education*, 36(3):211-224.
[2] http://www.bscs.org/library/BSCS_5E_Instructional_Approach_July_06.pdf
[3] Atkin, J. M. and Karplus, R. (1962). Discovery of invention? *Science Teacher* 29(5): 45.
[4] e.g. http://www.science.org.au/primaryconnections/constructivist.htm

METHOD OF SUPERPOSITION

10. <u>Principle</u>: **Statically indeterminate thermal loading**

Engage

Use BBC News report at http://news.bbc.co.uk/1/hi/wales/north_west/3125813.stm and http://news.bbc.co.uk/1/hi/uk/3126441.stm to illustrate very real problem of thermal expansion in train tracks.

Right picture from http://www.volpe.dot.gov/sdd/buckling.html (public information from RITA)

Explore

Long slender components subject to compressive loads usually show a preference to fail buckling rather than by material failure. A plastic ruler is an easy example. This can also happen in railway tracks. The rail is stretched before laying to reduce expansion in hot weather; but at very high temperatures rails can expand so much that they buckle. The heat alone can cause buckling but so can the passage of a train over the rail. To reduce the risk of buckling speed restrictions are imposed on high speed lines from about lunchtime until early evening.

Explain

Consider the stress and load induced by the temperature changes mentioned above. For example in a rail laid at 20ºC so that it is stress-free the compressive stress induced by a 30ºC temperature rise is given by:

$$\sigma = \varepsilon E = \alpha(\Delta T)E$$

Where ε is the strain induced, E is the Young's modulus of the rail (≈ 206GN/m^2), ΔT the temperature rise and α the coefficient of thermal expansion (for rail steel ≈ 12×10^{-6} per ºC). So,

$$\sigma = 12\times10^{-6}\times(30)\times206\times10^{9} = 74 \text{ MPa}$$

If the cross-section area of the rail is 14.5cm^2 then the buckling load is about

$$P = \sigma A = 74\times10^{6}\times14.5\times10^{-4} = 108\,\text{kN}$$

Elaborate

The free expansion of the rail if it was 25m long would be:

$$\delta = \alpha L \Delta T = 12 \times 10^{-6} \times 25 \times 30 = 9\,\text{mm}$$

So, if a 4mm expansion gap is present then the stress will be reduced:

$$\sigma = \varepsilon E = \frac{(9-4) \times 10^{-3}}{27} \times 206 \times 10^{9} = 41.2\,\text{MPa}$$

A reduction of 44%. However modern welded tracks have very few expansion gaps in order to give a smooth ride and hence are more susceptible to buckling at high temperatures. If rails were pre-tensioned to avoid buckling in summer then they would be more susceptible to cracking in winter under high tensile loads.

Evaluate

Ask students to attempt the following examples:

<u>*Example 10.1*</u>

A jewellery pendent is to be made by fitting a gold ring 12mm wide over a platinum ring 12mm wide at 1020ºC. The discs fit snugly inside one another at the elevated temperature with a common diameter of 50mm and so will lock together on cooling to room temperature. Both rings are 12mm thick. Calculate the radial pressure at the interface and the circumferential stress set-up in both rings at 20ºC.

Material Properties (http://www.goodfellow.com/csp/active/gfPeriodic.csp?form=All)

	α (/ºC)	E (GPa)	σ_y(MPa)
Gold	14×10^{-6}	78.5	205
Platinum	9×10^{-6}	170	185

<u>Solution:</u>

On cooling from 1020ºC

Change in circumference in gold = Change in circumference in platinum

Since gold and platinum have different coefficients of thermal expansion an interference fit will be set-up with each exerting an equal and opposite force on one another:

$$F_{Au} = F_{Pt} \qquad \text{(i)}$$

These circumferential forces together with the thermal contractions will cause the changes in circumferences, i.e.

$$(\alpha_{Au} \times \Delta T \times L_{Au}) - \frac{F_{Au} L_{Au}}{A_{Au} E_{Au}} = (\alpha_{Pt} \times \Delta T \times L_{Pt}) + \frac{F_{Pt} L_{Pt}}{A_{Pt} E_{Pt}} \qquad \text{(ii)}$$

Substitute (i) in (ii) and:

$$F_{Au} = \frac{\Delta T(\alpha_{Au} - \alpha_{Pt})}{\frac{1}{A_{Au}E_{Au}} + \frac{1}{A_{Pt}E_{Pt}}} = \frac{500(14-9)\times 10^{-6}}{\frac{1}{(144\times 10^{-6})(78.5\times 10^{9})} + \frac{1}{(144\times 10^{-6})(170\times 10^{9})}} = 19.9\,\text{kN}$$

Circumferential stresses are

$$\sigma_{Au} = \sigma_{Pt} = \frac{F_{Au}}{A_{Au}} \frac{19.9\times 10^{3}}{144\times 10^{-6}} = 134\,\text{MPa}$$

Also Circumferential stress, $\sigma_{circ} = \frac{pr}{t}$ from thin-walled pressure vessel theory so

$$p = \frac{\sigma t}{r} = \frac{134\times 10^{6}}{} \quad p = \frac{\sigma t}{r} = \frac{134\times 10^{6}\times 12\times 10^{-6}}{50\times 10^{-6}} = 32\,\text{MPa}$$

Could argue that thin-walled pressure vessel assumption does not apply because *r/t* = 4.166 which is not greater than 10.

Example 10.2

Ask students to look for two other examples in their everyday life and explain how the above principles apply to each example.