EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F5: Momentum

This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

CONTROL VOLUME ANALYSIS

5. Topic: **Momentum**

Engage:

Take a hairdryer, stand and clamp, some balloons, and paperclips into class. Set up the hairdryer with the stand and clamp so that it is blowing vertically up on its cold setting. Blow up a balloon and place it in the air-stream and it will probably be pushed up to the ceiling.

Explore:

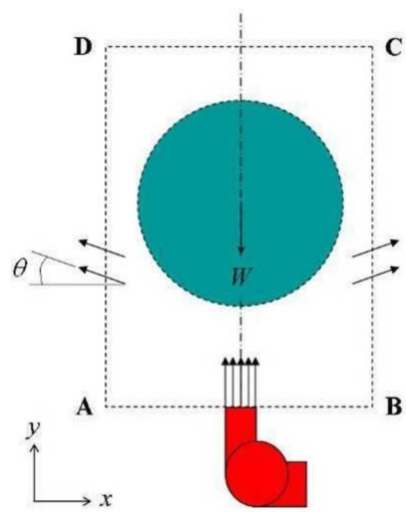
Capture the balloon and attach a paperclip to the neck of the balloon and again place it in the airstream. As you add more paperclips the balloon will float lower and lower in the airstream. To save time you could use a series of balloons inflated to the same diameter with various numbers of paperclips attached. There is a nice video of school kids performing this experiment[4].

Explain:

Explain that when the balloon hovers at a stable height, based on Newton's second law applied to the balloon, the force on the balloon from the jet of air is equal to the weight of the balloon.

We can draw a cylindrical control volume around but not including the balloon and equate the change in momentum to the external forces on the control volume, i.e.

$$\sum \mathbf{F} = \dot{m}(\mathbf{v}_2 - \mathbf{v}_1)$$



In this case the external force is the weight of the balloon. Sometimes it is easier to consider three scalar components of this vector expression, i.e.

$$\sum F_x = \dot{m}\left(v_{2_x} - v_{1_x}\right)$$

$$\sum F_y = \dot{m}\left(v_{2_y} - v_{1_y}\right)$$

$$\sum F_z = \dot{m}\left(v_{2_z} - v_{1_z}\right)$$

4 www.planet-scicast.com/view_clip.cfm?cit_id=2709

Elaborate:

Consider the cylindrical control volume around the balloon. Assume that the velocity of the airstream leaving the hairdryer is a uniform 50m/s and that all of air leaves the control volume at the same speed but at an angle, $\theta = 30°$ from the horizontal.

In practice this angle will vary from about zero close to the bottom of the balloon and approach 90° close to the horizontal diameter of the balloon.

The mass flow rate at entry, $\dot{m} = \rho_{air} A v = 1.2 \times \frac{\pi}{4} \times 0.06^2 \times 50 = 0.17\,\text{kg/s}$

Assuming the hairdryer has a circular nozzle with an outlet diameter of 6cm. The momentum of the fluid crossing AB will be

$$\dot{m}v_{AB} = 0.17 \times 50 = 8.48\,\text{N}$$

And in the *y*-direction for cylindrical surface, given that continuity demands the total mass flow across the surface equals that across AB,

$$\dot{m}v_{AD} \sin\theta = 0.17 \times 50 \sin 30 = 4.24\,\text{N}$$

So, applying the momentum equation,

$$W = \dot{m}v_{AB}(1 - \sin 30) = 4.24\,\text{N}$$

This hair dryer could keep a balloon and tail of paperclips weighing 4.24N (=0.42kg) in the air. The airstream is slowed down by entrapment of the surrounding air and so the nozzle velocity will not be achieved at all heights above the hairdryer, which is why the balloon sinks down as it is loaded with paperclips.

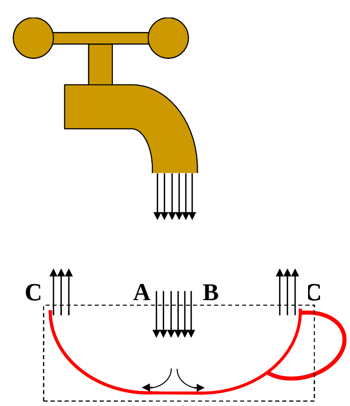
Evaluate:

Invite students to attempt the following examples:

<u>*Example 5.1:*</u>

Calculate the force exerted by the water from the tap in your kitchen sink on a coffee cup held in the flow when the tap is open to its maximum.

<u>Solution:</u>



Estimate the water flow by how long it takes to fill a measuring jug. In the editor's kitchen it took 5 seconds to fill a 1 liter measuring jug, so the mass flow rate is

$$\dot{m} = \frac{\rho_{H_2O} V}{T} = \frac{1000 \times 0.001}{5} = 0.2\ \text{kg/s}$$

The diameter of the orifice in the tap is 17mm and so the velocity of the flow is

$$v = \frac{\dot{m}}{\rho A} = \frac{0.2}{1000 \times \left(\frac{\pi \times 0.017^2}{4} \right)} = 0.88\,\text{m/s}$$

Hence, the momentum of the water leaving the tap at AB is

$$\dot{m}v_{AB} = 0.2 \times 0.88 = 0.176\,\text{N}$$

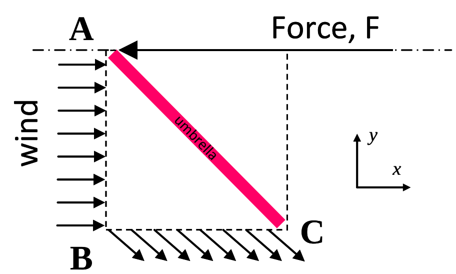
The water is turned through 180° in the cup and so emerges with a momentum of equal and opposite sign so that the total change in momentum is $2\dot{m}v_{AB} = 0.352\,\text{N}$.

This is about one third of the weight of the editor's best porcelain coffee cups.

Example 5.2

Calculate the force required to hold an umbrella against a 30mph wind. Assume the wind is blowing horizontally so you are holding your umbrella head into wind, i.e., the handle is parallel to the wind direction and the umbrella deflects the wind to leave at an angle of 45°. The umbrella is symmetrical about the handle with a projected area perpendicular to the wind, which is a circle of diameter 1.2m.

Solution:



Consider a control volume such that wind enters at AB with a velocity of 13.4 m/s (≡ 30mph) and leaves at 45° to this along BC. We can apply the momentum equation in the direction of the wind such that

$$\sum F_x = \dot{m}\left(v_{2_x} - v_{1_x}\right)$$

and $F = -\rho_{air} A v_{1_x} \left(v_{1_x} \cos 45 - v_{1_x}\right) = -1.29 \times \frac{\pi 1.2^2}{4} \times 13.41^2 \left(\cos 45 - 1\right) = 76.8\,\text{N}$

If you were walking into the wind, then it would be necessary to construct velocity diagrams for the wind velocity at entry and exit to, what would become, a moving control volume.