EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F4: Dynamics of fluid motion

This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

FLUIDS IN MOTION

4. Topic: **Dynamics of fluid motion**

Engage:

Take a hair dryer and some table tennis balls into class. Invite a couple of pairs of students to turn on the hair dryer, point it upwards and balance a ball in the air-stream. Ask them to describe what they feel when they try to remove the ball from the air-stream (a force holding the ball in the air-stream). There is a good video and explanation of this experiment[4].

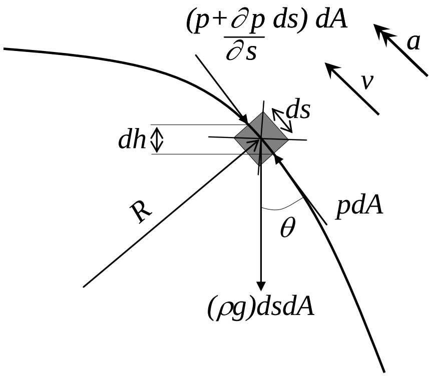
Explore:

Show a video of flow around a tennis ball with vortex shedding, by searching in Youtube using the words "*Fluid Mechanics - Cool science experiment*" [5], and discuss the presence of areas of laminar and turbulent flow with vortices being formed downstream of the ball. Vortices are low pressures zones formed downstream of blunt objects and the ball tends to move towards the low pressure zone which is why the table tennis ball bounces around in the flow from the air dryer.

Explain:

Return their attention to the video and the laminar flow region where the flow visualization with the smoke represents streamlines. Using a Lagrangian approach, consider a particle of fluid on a streamline and apply Newton's second law in the direction of the streamline, i.e. resolve forces,

$$pdA - \left(p + \frac{\partial p}{\partial s} ds \right) dA - (\rho g) ds dA \cos\theta = \rho a_s ds dA$$



where a_s is the acceleration of the particle along the streamline. For two points on the streamline and assuming no viscous forces, it can be shown that

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} + gz_1 = \frac{v_2^2}{2} + \frac{p_2}{\rho} + gz_2 = gH$$

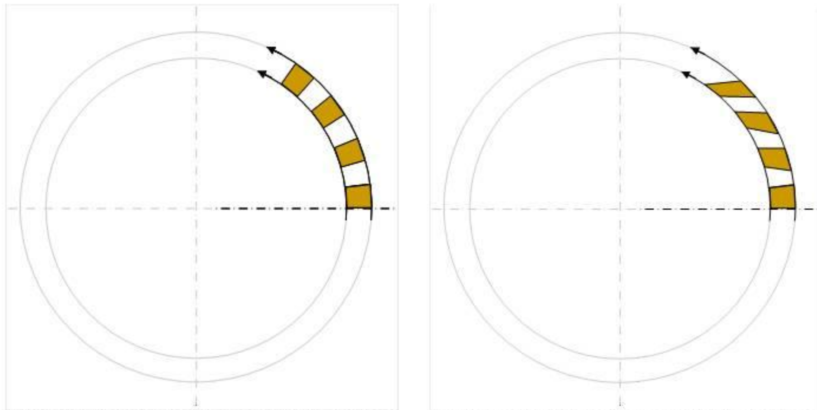
which is known as Bernoulli's equation and Bernoulli's constant, H is constant along a streamline.

Note that Bernoulli's equation can be expressed such that the terms have the units of pressure by normalizing with density, ρ; or of length or head by normalizing with gravitational acceleration, g. Thus, piezometric head can be defined as $(p/g\rho + z)$; total head as $(v^2/2g + p/g\rho + z)$; and stagnation or total pressure as $(p + \rho v^2/2)$.

[4] www.efluids.com/efluids/gallery_exp/exp_pages/hairdryer.jsp
[5] www.youtube.com/watch?v=7KKFtgx2anY

Elaborate:

Highlight that a vortex consists of a system of concentric circular streamlines in which the fluid velocity is inversely proportional to the radius, i.e. *vr* = constant. When Bernoulli's constant, H is the same throughout the vortex it is known as a free vortex with rotational flow in which fluid elements rotate in circles but do not deform.



Rotational flow without element deformation   Irrotational flow with element deformation

In a free vortex, Bernoulli's constant requires the pressure to drop as the velocity rises towards the center of a vortex. The pressure drop causes:

- free surfaces to curve such in the bath around an open plug-hole;
- water vapor to condense, which generates the vapor streams behind aircraft as vortices shed from the wings;
- low pressure in the center of tornadoes and hurricanes so that a secondary airflow is established along the ground towards the center and then up into the core.

In practice, at the center of most naturally occurring free vortices the viscosity of the fluid causes it to rotate as a solid body creating a compound or Rankine vortex.

A category five hurricane (e.g. Katrina in 2005) has wind speeds in excess of 155 mph. When it is over the sea, the outer portion can be modeled as a free vortex so if the maximum wind speed of 160mph (71.5m/s) occurs at 1 mile from the center then the wind speed at 5 miles from the center will be given by

$$vr = C \quad \text{so} \quad \frac{v_5}{v_1} = \frac{r_1}{r_5} = 0.2 \quad \text{and} \quad v_5 = 71.5 \times 0.2 = 14.3\,\text{m/s (32mph)}$$

Applying Bernoulli's equation through the hurricane

$$p_1 + \rho \frac{v_1^2}{2} = p_5 + \rho \frac{v_5^2}{2}$$

So the pressure difference between the two locations at the same height ($z_1 = z_5$) is

$$p_5 - p_1 = \frac{\rho}{2}\left(v_1^2 - v_5^2\right) = \frac{1.29}{2}\left(71.5^2 - 14.3^2\right) = 3165\text{Pa}$$

This is a large pressure difference and helps to explain the catastrophic damage caused by a category five hurricane, which is the most intense category of storms.

Evaluate:

Invite students to attempt the following examples:

Example 4.1

Calculate the force on your hand when you hold it as far out of the car window as you can reach, palm against the wind, when the car is travelling at 70mph.

Solution:

Measure the area of your hand. For the editor's hand, the area is approximately 0.018m^2.

Apply Bernoulli's equation for point ① some distance in front of your hand and for point ② just in front of your palm such that both points are on the same streamline,

$$\rho\frac{v_1^2}{2} + p_1 + \rho g z_1 = \rho\frac{v_2^2}{2} + p_2 + \rho g z_2$$

The air velocity at point ① is 70mph (~31m/s = v_1) and zero at your palm (v_2 =0). The gage pressure at point ① will be zero (p_1=0) and there is no height difference between the points if you are on a level road so $z_1 = z_2$. Thus, with ρ_{air}= 1.29 kg.m^{-3}

$$p_2 = \rho_{air}\frac{v_1^2}{2} \quad \text{so} \quad p_2 = \rho_{air}\frac{v_1^2}{2} = 1.29\frac{31^2}{2} = 620\,\text{Pa}$$

Now, $F = pA = 620 \times 0.018 = 11.2\,\text{N}$

Example 4.2

If a vacuum cleaner hose is to be able to pick-up a layer of dust 0.5mm thick from a height of 3cm, calculate the maximum average velocity that needs to be achieved at the inlet to the hose.

Solution:

Let's assume the dust is dry sand for which density data is available, i.e. ρ = 1600 kg/m^3.

If the cross-section area of the hose is *A*, then the force required to lift the area of sand covered by the hose is

$$F = mg = \rho A t g \quad \text{and the pressure at the hose,} \quad p = F / A = \rho t g$$

where *t* is the thickness of the layer of sand. Applying Bernoulli's equation for a point on the surface of the sand ① where the static pressure and velocity are zero and at the hose ② so

$$\rho g h_1 = \rho\frac{v_2^2}{2} + p_2 + \rho g h_2$$

or $$v_2 = \sqrt{2g(h_1 - h_2) - 2tg} = \sqrt{(2\times 9.81\times 0.03) - 2\times 0.0005\times 9.81} = 0.76\,\text{m/s}$$