# EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

## F3: Kinematics of fluid motion

*This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".*

**INTRODUCTION**

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

**FLUIDS IN MOTION**

3. Topic**: Kinematics of fluid motion**

**Engage:**

Take a flask of coffee, some cream or milk, a glass, and some paper cups into class. Offer a free cup of coffee to students as they come into class and ask them to add the cream slowly and watch what happens. Repeat the process yourself except use the glass so that the students can see the mixing process in three-dimensions.

**Explore:**

Show a video of *How to Cream Coffee*[4] (search YouTube using the italicized words). Explain that the cream shows us 'streaklines'. A streakline is an instantaneous locus of all particles originating from a common point. You might also like to show the video *coffee and cream mix slow motion*[5].

Show a video of the *flow around a sphere (inH20)* in a water tank in which red dye is introduced on the upstream side of the sphere[6].

Explain that the fluid dye shows us 'streaklines' in the same way as the cream in the coffee. Define a pathline as a history of particle's location, and lines that are tangential to the velocity vectors of particles in the flow as streamlines. An example of streamlines would be the lines left in a time-lapsed photograph by the headlights of a moving car.

For steady-flow streaklines, pathlines and streamlines are coincident. You could use a video of flow in front of and behind a *BMW 3series Convertible 1987 in the wind tunnel*[7] to illustrate the difference between steady and unsteady flow. You could talk about the distance between streamlines representing pressure and the relationship with increasing and decreasing velocity.

**Explain:**

Highlight that when watching the coffee and cream or the videos our eyes tend to follow the movement of the cream (or the smoke) in the flow and that this can be equated to the Lagrangian approach to fluid mechanics.

Show the BMW convertible video again and use your laser pointer or the cursor on the screen to highlight a single point. Explain that considering the fluid motion at a single point in this way is an Eulerian approach to fluid mechanics.

[4] www.youtube.com/watch?v=9EsZ7GSUcI4&NR=1
[5] www.youtube.com/watch?v=M4vB6kAhuN0
[6] www.youtube.com/watch?v=4zHIjyj-vEo
[7] www.youtube.com/watch?v=XS3sbYJHkSw&feature=channel

**Elaborate**

Continue the Eulerian approach and instead of considering a single point, expand it to a box, through which the fluid is flowing. Explain that such a box is known as a control volume in fluid mechanics. Discuss that in steady flow, the amount of fluid entering and leaving the control volume at any instant in time is the same, i.e., the mass of the fluid in the box is constant, which leads to the continuity equation,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \text{constant} = \dot{m}$$

where $\rho$ is the fluid density, $v$ the fluid velocity and $A_1$ and $A_2$ are cross-section areas at the entry and exit from the control volume, and usually considered to be connected by pathlines, i.e., lines across which no particles flow.

In unsteady flow the mass of fluid in the control volume changes with time and hence the continuity equation has to be modified as

$$\frac{dm_{cv}}{dt} + \dot{m}_{out} + \dot{m}_{in} = 0$$

where the first term represents the accumulation of fluid in the control volume and the last two terms represent the outflow and inflow rates across the surfaces of the control volume.

Use the continuity equation to calculate the flow rates for the inlet and outlet ducts of the air conditioning in the classroom. It is recommended that the air in a classroom should be completely changed 12 times per hour[8].

Estimate the volume of the classroom based on floor dimensions of 35ft (10.7m) by 25ft (7.6m) and a ceiling height of 8ft (2.4m),

$$V = 10.7 \times 7.6 \times 2.4 = 195.2\,\text{m}^3$$

So the volumetric flow rate required is $0.65\text{m}^3/\text{s}\left(= (195.2 \times 12)/(60 \times 60)\right)$ and if we assume constant air density then

$$A_1 v_1 = A_2 v_2 = 0.65\text{m}^3/\text{s}$$

If the room has five inlet vents of dimensions 3ft (0.914m) by 6in (0.153m) and a 2ft (0.610m) square outlet vent then

$$v_1 = \frac{0.65}{(0.914 \times 0.153) \times 5} = 0.93\,\text{m/s}$$

$$v_2 = \frac{0.65}{0.610^2} = 1.75\,\text{m/s}$$

**Evaluate**

Invite students to attempt the following examples:

[8] http://ateam.lbl.gov/Design-Guide/DGHtm/roomairchangerates.htm

*Example 3.1*

It is recommended that bathrooms of 100sq.ft. or less should have a ventilation of 1 CFM per sq.ft of area and no less than 50CFM[9]. This is usually achieved with a small fan that extracts air. Estimate the volumetric flow rate for your bathroom and the velocity of air flowing under the closed door to replace the air extracted by a fan.

Solution:

The editor's bathroom is 8ft by 6ft hence

$$\text{Volumetric flow rate} = \text{Floor area} \times 1 = 8 \times 6 = 48\,\text{CFM} \equiv 0.023\ \text{m}^3/\text{s}$$

If the door is 3ft wide (0.9m) with a half inch (0.0127m) gap underneath it then the velocity of the flow will be given by

$$v = \frac{0.023}{0.9 \times 0.0127} = 2\,\text{m/s}\ (\sim 4.5\ \text{mph})$$

*Example 3.2*

If a car tire with a slow puncture takes three days (72 hours) to decrease from its recommended pressure of 35psi to 22psi at which the low pressure light activates on the instrument panel, calculate the area of the hole, $A$, given the mass flow rate, $\dot{m}$, from the tire is given by

$$\dot{m} = \frac{0.66\,pA}{\sqrt{RT}}$$

where $R$ is the gas constant, $T$ is the absolute temperature and $p$ is the tire pressure. You may assume that the change in volume of the tire is negligible.

Solution:

Using the continuity equation:

$$\frac{dm_{cv}}{dt} + \dot{m}_{out} + \dot{m}_{in} = 0$$

so $$\frac{d\rho V_{cv}}{dt} + 0.66\frac{pA}{\sqrt{RT}} + 0 = 0$$

Applying the ideal gas law

$$V_{cv}\frac{d}{dt}\left(\frac{p}{RT}\right) + 0.66\frac{pA}{\sqrt{RT}} = 0$$

assuming A, R (=587 J/kgK), T and V are constant and rearranging

$$dt = \frac{1.52V}{A\sqrt{RT}}\frac{dp}{p}$$

Integrating and applying initial and final pressures as limits

[9] www.hvi.org/resourcelibrary/HowMuchVent.html

$$t = \frac{1.52V}{A\sqrt{RT}} \ln \frac{p_i}{p_f}$$

Now, the volume of the tire can be calculated by assuming the tire and wheel to be concentric cylinders of outside diameters 0.6m and 0.4m respectively and thickness (= *l*) 0.18m (based on measurements taken from the editor's car), thus

$$V = \frac{\pi}{4}\left(d_{tire}^2 - d_{wheel}^2\right) \times l = \frac{\pi}{4}\left(0.6_{tire}^2 - 0.4_{wheel}^2\right) \times 0.18 = 0.028\,\mathrm{m}^3$$

So, if the temperature is assumed to be 21°C then

$$A = \frac{1.52V}{\Delta t\sqrt{RT}} \ln \frac{p_i}{p_f} = \frac{1.52 \times 0.028}{(72 \times 60 \times 60)\sqrt{587 \times 300}} \ln \frac{35}{22} = 1.82 \times 10^{-10}\,\mathrm{m}^2$$

Assuming a round hole, it would have a diameter of $1.5 \times 10^{-5}$ (≡ 15μm) since

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 1.82 \times 10^{-10}}{\pi}} = 1.5 \times 10^{-5}\,\mathrm{m}$$