EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F2: Statics

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This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

INTRODUCTORY CONCEPTS

2. Topic: **Statics**

Engage:

Take a basin of water and some apples into class. Invite a couple of students to do some apple bobbing, i.e. trying to grab the apples with their teeth. You should probably take some towels into class too!

Explore:

Ask the students who do some apple bobbing and observe how the apples behave. If you can get hold of some small applies then you could pass around an apple in a cup of water and students can experiment for themselves. Discuss the following with the students:

(i) when pushed down an apple experiences an upward force restoring it to the surface; and

(ii) that most apples have a stable orientation.

Explain:

Remind the students that Archimedes principle defines the buoyancy force as equal to the weight of the volume of water displaced,

$$F_B = (\rho g V)_{\text{displaced fluid}}$$

Note that, sometimes $\gamma = \rho g$ is used and is known as the specific weight of the fluid. So to keep an apple submerged it is necessary to apply a force,

$$S = mg - F_B$$

where mg is the weight of the apple.

When it is submerged, the apple experiences a force acting on its skin due the pressure of the water. This force on the curved surface of the skin must act through the center of curvature of the surface. The pressure is related to the depth in the downward direction, $-dz$ and density of the water, ρ such that

$$dp = -\rho g \cdot dz$$

It would be appropriate to explain that absolute pressure is measured relative to absolute zero pressure which could only occur in a perfect vacuum; however it is common engineering practice to measure pressure relative to atmospheric pressure. A pressure measured in this way is known as gage pressure. Standard atmospheric pressure is defined as 14.7psi or 101kPa at sea-level.

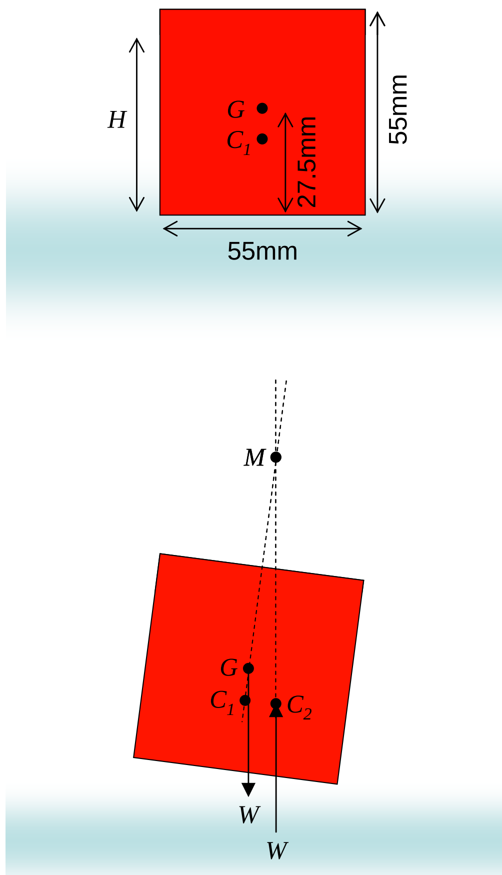
Elaborate:

To elaborate on the stability of the apples it is preferable to simplify the situation to a cylinder of uniform material of specific weight, γ_{cyl} = 9000 N/m3, i.e. slightly less than water, and of length and diameter 55mm. If we consider force equilibrium in the vertical direction for the cylinder floating with its axis vertical, then the weight equals the buoyant force, i.e.

$$\sum F_y = 0$$

and $$mg = \gamma_{H_2O} \pi r^2 H$$

or $$\rho_{cyl} \pi r^2 L g = \gamma_{H_2O} \pi r^2 H$$



where H is the submerged depth, L is the length and r the radius of the cylinder.

$$H = \frac{\gamma_{cyl} L}{\gamma_{H_2O}} = \frac{9000}{9810} \times \left(55 \times 10^{-3}\right) = 50.5 \times 10^{-3}\,\text{m}$$

Now consider the stability of the cylinder. The buoyancy force, W, acts through the centroid of the displaced volume of fluid, C_1, which when the cylinder is rotated by an external force moves to C_2. The weight of cylinder, W, always acts at its centroid, G, so that the two forces W form a couple which tends to return the cylinder to its original position if the original was stable or tends to rotate it further if the original position was unstable.

The lines joining the centroid of the cylinder, G, to the centroids of the displaced volume of water, C_1 and C_2, intersect at the metacenter, M. It can be shown that the distance C_1M is given by

$$\overline{C_1 M} = \frac{I_O}{V}$$

where I_O is the second moment of area of the water-line section about the axis through its centroid and V is the submerged volume. Alternatively, given from the geometry that

$$\overline{C_1 G} + \overline{GM} = \overline{C_1 M}$$

then $$\overline{GM} = \frac{I_O}{V} - \overline{C_1 G}$$

When the distance from the centroid of the cylinder to the metacenter, $\overline{GM}$ is positive, the cylinder is stable.

So for the apple idealized as a cylinder,

$$I_O = \frac{\pi d^4}{64} = \frac{\pi \times \left(55\times 10^{-3}\right)^4}{64} = 4.49\times 10^{-7}\,\mathrm{m}^4$$

and, from the diagram

$$\overline{C_1G} = 27.5\times 10^{-3} - \frac{50.5\times 10^{-3}}{2} = 2.25\times 10^{-3}\,\mathrm{m}$$

so $$\overline{GM} = \frac{I_O}{\frac{\gamma_{cyl}}{\gamma_{H_2O}}\frac{\pi d^2 L}{4}} - \overline{C_1G} = \frac{4.49\times 10^{-7}}{\frac{9000}{9810}\frac{\pi \times \left(55\times 10^{-3}\right)^3}{4}} - 2.25\times 10^{-3} = 0.00149\mathrm{m}$$

Thus, the apple is stable in this orientation, with its axis of symmetry or stalk vertical.

Evaluate:

Invite students to attempt the following examples:

Example 2.1

Calculate the force needed to push up the 4cm diameter plug in a bath of water 20cm deep, to let the water out, if the plug is a loose-fit in the plug-hole and the lever mechanism that operates the plug provides a mechanical advantage of 3.

Solution:

Pressure on the plug,

$$p = \rho_{H_2O} g z = 1000\times 9.81\times 0.2 = 1962\,\mathrm{N/m}^2$$

assuming atmospheric pressure on the surface at the surface of the bath water and in the drain under the plug.

Force on plug, $F = pA = p\frac{\pi d^2}{4} = 1962\frac{\pi \times 0.04^2}{4} = 2.46\,\mathrm{N}$

With a mechanical advantage of 3 the force required on mechanism will be 0.82N (=2.46/3).

Example 2.2

When a car slides off the road into a river, which is 2.5m deep, it lands on its side at 45 degrees to the vertical such that the passenger side doors are jammed into the soft riverbed. Calculate the force required to open the driver's door if the car remains watertight and the bottom of the door is just touching the riverbed. Assume the door is a rectangle of height 1.2m and width 1.1m weighing 33kg with the handle on the opening edge.

Solution:

It can be shown that the force acting on an area, $F = p_C A$

where p_C is the pressure at the centroid on the area, A. In this case the centroid is located at $(L/2)\cos 45°$ above the riverbed where L is the height of the car door. Hence, the distance from the water surface to the centroid of the door,

$$h = 2.5 - \frac{1.2}{2}\cos 45° = 2.08\,\mathrm{m}$$

Thus, the pressure, $p = \rho_{H_2O} gz = 1000 \times 9.81 \times 2.08 = 20{,}400\mathrm{Pa}$

And, the force acting on the door, $F = p_C A = 20363 \times (1.2 \times 1.1) = 26{,}900\mathrm{N}$

This force will act at the center of pressure, which will be on the vertical center-line of the door but below the centroid due to the pressure gradient. Assuming the force to open the door, F_{open} is applied approximately level with the center of pressure, then taking moments about the hinge,

$$\frac{Fw}{2} + \frac{mgw}{2} - F_{open} w = 0$$

where w is the width of the door, thus

$$F_{open} = \frac{F}{2} + \frac{mg}{2} = \frac{26879}{2} + \frac{33 \times 9.81}{2} = 13601N$$

This is a huge force. It is probably a false assumption that the car will be watertight; but nevertheless, if your car is immersed in an accident it will be extremely difficult, if not impossible, to open a door.[♣]

Example 2.3

In your car you have a coffee cup of diameter 9cm at the rim and it is full to within 10mm of the rim. When the cup is placed in a cup-holder, how fast can you accelerate without spilling your coffee?

Solution:

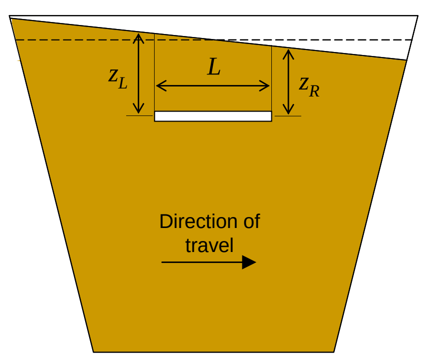
Pressure in a fluid is ρgz

so the force on the left end of the element of fluid in the diagram will be

$$F_L = \rho g z_L A$$

and for the right end, $F_R = \rho g z_R A$

so the net force in the direction of travel is



[♣] In these circumstances it is suggested that you should exit through a window. So, your first action on realizing you are going to impact with the water should be to open a window since the electrics will fail quickly on immersion. If you need to break a window, hit it in a corner with a sharp object.

$$F = \rho g A(z_L - z_R)$$

and this must be equal to force due to the acceleration, i.e. $F = ma$ thus substituting for the mass of the element

$$\rho g A(z_L - z_R) = \rho A L a$$

or $$a = \frac{g(z_L - z_R)}{L}$$

and using similar triangles $a = \dfrac{9.81 \times 10 \times 10^{-2}}{9 \times 10^{-2}/2} = 2.45\,\mathrm{m/s^2}$

This equivalent to 0 to 16mph in 3 seconds and is why we put lids on our coffee (or don't fill your cup too near the top)!