EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F11: Compressible flow

This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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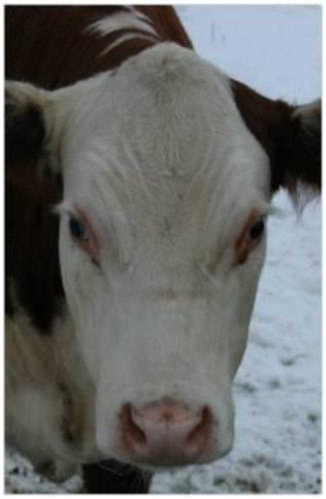
[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

FLOW

11. Topic**: Compressible flow**

Engage:



Show a news-clip entitled '*Bull leaps into crowd in Tafalla Northern Spain 30 hurt*' by searching for this title on YouTube[4]. Run the clip once (it is only 29 secs) and then a second time pausing it at about 6 seconds and discuss how the crowd is reacting to the bull. They are moving away; the bull is slowed down by the fence and so people have time to get out of the way. The speed of signaling (about the danger) through the crowd could be considered to be faster than the speed of the bull.

Then let the clip[5] run to about 10 seconds and pause it again. Discuss the difference in crowd behavior. Now the bull has got up some speed and the people in the crowd cannot react faster enough so the bull runs into them. The speed of signaling through the crowd could be considered to be slower than the speed of the bull.

If you feel students need to be convinced that crowds can behave in an analogous way to fluid flow, then you might find the following videos useful:

a. a flock of sheep flowing around a stationary car (search on YouTube for '*A sheep dog herds sheep near Wanaka, New Zealand*'[6]); and

a flock of sheep parting to allow a car through which is analogous to sub-sonic flow with some signaling moving through the flock faster than the motion of the car (search on YouTube for '*New Zealand in 56 secs*'[7]).

Explore:

Discuss how the signaling mentioned above is achieved in air with an airplane moving through it at subsonic speeds, i.e., that pressure disturbances caused by the plane's motion are propagated in all directions at the speed of sound and at some distance are attenuated by the viscosity of the air. Ahead of the plane they act as a signal to the air that the plane is coming and the air begins to move out of the way of the plane so that smooth flow is achieved around the plane. This can occur if the speed of the plane, v is slower than the speed of signaling, i.e., the speed of sound, c.

You can see this happening using smoke in a wind tunnel, e.g., search on YouTube for a video entitled '*How wings work? Smoke streamlines around an airfoil*'[8].

[4] www.youtube.com/watch?v=2ksQZmxI_Nw
[5] An alternative but longer video can be found at http://www.youtube.com/watch?v=SWTiHCK5ZpM or by searching on YouTube using '*Bull goes on rampage through crowd*'. The corresponding pause points are about 8 seconds and 12 seconds respectively.
[6] www.youtube.com/watch?v=sYJaMGcrtWE
[7] www.youtube.com/watch?v=WT3dk7HOi1A

Now, ask the students to consider what will happen if $v > c$?

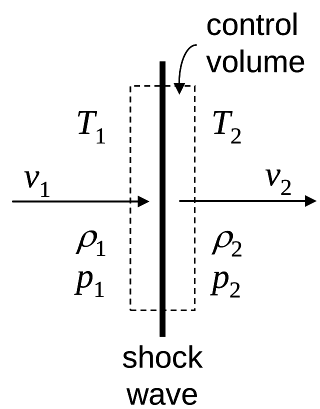
This is analogous to the bull running into the crowd. The air ahead of the plane has no warning and so does not move out of the way. The bull is brought to a stop by the group of compressed people and he moves off to one side; but the plane is not brought to a stop, so what happens?

A shock wave forms just ahead of the plane. Let us consider shifting our frame of reference to that of the pilot, i.e., a stationary plane with supersonic air moving towards it ($v > c$). As the air passes through the shock wave its velocity is reduced so that $v < c$ and the air has time to part and flow around the plane. This transition across the shock wave not only involves a deceleration of the air but also an increase in its static temperature, pressure and density. For a good illustration of this search on Youtube[9] for a video entitled: '*F18 Supersonic flyby with sonic boom on CVN69 USS Eisenhower*'.

Explain:

Explain that we can analyze the region around a shock wave by enclosing it in a control volume. Applying the momentum equation to the control volume,

$$p_1 A + \dot{m} v_1 = p_2 A + \dot{m} v_2$$



Continuity of mass flow requires,

$$\rho_1 v_1 A = \rho_2 v_2 A$$

so $$p_1 - p_2 = \rho_2 v_2^2 - \rho_1 v_1^2$$

Assuming an ideal gas both sides of the shock so that

$$\rho_{1,2} = \frac{p_{1,2}}{RT_{1,2}}$$

and substituting

$$p_1 - p_2 = \frac{p_2 v_2^2}{RT_2} - \frac{p_1 v_1^2}{RT_1} \quad \text{or} \quad p_1\left(1 + \frac{v_1^2}{RT_1}\right) = p_2\left(1 + \frac{v_2^2}{RT_2}\right)$$

Now defining the Mach number, M_a as $M_a = \frac{v}{c} = \frac{v}{\sqrt{kRT}}$ where $k = \frac{C_p}{C_v}$ so

$$\frac{p_1}{p_2} = \frac{\left(1 + k\left(M_a\right)_2^2\right)}{\left(1 + k\left(M_a\right)_1^2\right)}$$

Note that when M_a<0.3 incompressibility can be assumed.

Thus, since $\left(M_a\right)_2$>1 and $\left(M_a\right)_1$<1, p_2>p_1 and static pressure increases across the shock wave.

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8 www.youtube.com/watch?v=6UlsArvbTeo
9 www.youtube.com/watch?v=9lu9o6pCHlM

Assuming adiabatic conditions for the control volume so that the stagnation temperature is constant, i.e. $(T_0)_1 = (T_0)_2$ it can be shown that

$$\frac{T_2}{T_1} = \frac{T_2}{T_0} \times \frac{T_0}{T_1} = \frac{2 + (M_a)_1^2 (k-1)}{2 + (M_a)_2^2 (k-1)}$$

Thus, again, since $(M_a)_2 > 1$ and $(M_a)_1 < 1$, $T_2 > T_1$ and temperature increases across the shock wave.

Returning to mass continuity for the control volume,

$$\rho_1 v_1 A = \rho_2 v_2 A$$

And substituting $\rho_{1,2} = \frac{p_{1,2}}{RT_{1,2}}$ and $M_a = \frac{v}{c} = \frac{v}{\sqrt{kRT}}$ we can obtain

$$\frac{T_2}{T_1} = \left[\frac{p_2 (M_a)_2}{p_1 (M_a)_1} \right]^2$$

Equating this with the previous expression for the temperature ratio we can obtain

$$(M_a)_2 = \sqrt{\frac{2 + (k-1)(M_a)_1^2}{2k(M_a)_1^2 - (k-1)}}$$

Elaborate:

Introduce the idea that can produce supersonic flow for testing shapes in a wind tunnel using a convergent-divergent nozzle, known as a Laval nozzle, connected to a large reservoir of gas for which p_0, ρ_0, T_0 and v_0 are zero. Highlight that the Laval nozzle is the same shape as the exit of a rocket engine.

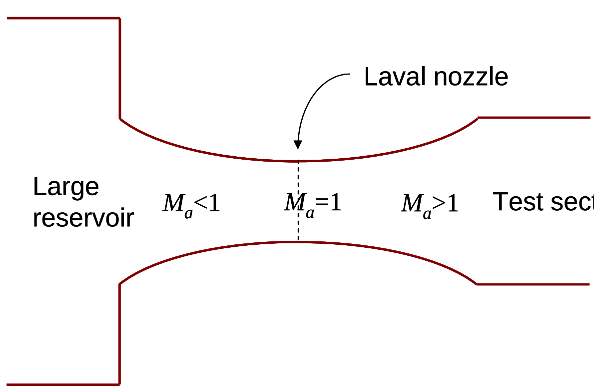
Applying Bernoulli's equation for any section of the nozzle,

$$\frac{v^2}{2} = \left(\frac{k}{k-1} \right) R (T_0 - T)$$

And normalizing by $c = \frac{v}{M_a} = \sqrt{kRT}$

$$\frac{v^2}{c^2} = \left(\frac{2}{k-1} \right) \left(\frac{T_0}{T} - 1 \right) = M_a^2$$

so $$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2} \right) M_a^2$$



and, recall that

$$\frac{T_2}{T_1} = \frac{T_2}{T_0} \times \frac{T_0}{T_1} = \frac{2 + (M_a)_1^2 (k-1)}{2 + (M_a)_2^2 (k-1)}$$

hence, for conditions at the throat, t $\quad \frac{T_t}{T} = \frac{T_t}{T_0}\cdot\frac{T_0}{T} = \frac{2+(k-1)M_a^2}{k+1}$

Now, for isentropic flow, $\quad \frac{T_0}{T} = \left(\frac{p_0}{p}\right)^{(k-1)/k} = \left(\frac{\rho_0}{\rho}\right)^{(k-1)}$

and hence $\quad \frac{\rho_t}{\rho} = \left[\frac{2+(k-1)M_a^2}{k+1}\right]^{1/k-1}$

Applying mass continuity for the nozzle, $\quad \rho v A = \rho_t v_t A_t$

so with $M_a = \frac{v}{c} = \frac{v}{\sqrt{kRT}}$ we can obtain $\quad \frac{A_t}{A} = \frac{\rho}{\rho_t}\left(\frac{T}{T_t}\right)M_a$

And substituting the values for the density and temperature ratios,

$$\frac{A_t}{A} = M_a\left[\frac{k+1}{2+(k-1)M_a^2}\right]^{k/(k-1)}$$

i.e., for k =1.4 and a Mach number of 3, the ratio of the throat diameter to the section diameter has to be

$$A_t = M_a\left[\frac{k+1}{2+(k-1)M_a^2}\right]^{k+1/2(k-1)} A = 3\left[\frac{2.4}{2+(0.4\times 3^2)}\right]^{1.4/0.8} A = 0.68A$$

Evaluate:

Invite students to attempt the following examples:

<u>*Example 11.1*</u>

In a laboratory, if a supersonic wind tunnel has a Laval nozzle with a throat area of 3cm^2 and is supplied from a large reservoir at a pressure of 20kPa and temperature 20°C, calculate the mass of air required to run the tunnel for 30secs.

<u>Solution:</u>

The mass flow rate through the throat will be given by

$$\dot{m} = \rho_t A_t v_t$$

and in the throat M_a =1 so $v_t = \sqrt{kRT_t}$ also $\frac{T_t}{T} = \frac{2}{\gamma+1}$ and $\frac{\rho_t}{\rho} = \left[\frac{2+(k-1)}{k+1}\right]^{1/k-1}$ so

$$\dot{m} = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} A_t\sqrt{\frac{2kRT_0}{k+1}}$$

and substituting the ideal gas equation, $\rho_0 = \frac{p_0}{RT_0}$

$$\dot{m} = \frac{A_t p_0}{\sqrt{T_0}} \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}} = \frac{\left(3\times10^{-6}\right)\times\left(2\times10^{4}\right)}{\sqrt{293}} \sqrt{\frac{1.4}{287}\left(\frac{2}{2.4}\right)^{(2.4)/0.8}} = 1.86\times10^{-4}\ \text{kg/s}$$

So for 30s we will need 0.0056kg or about 0.0067m^3 (=0.0056×1.2).

Example 11.2

An explosion occurs in a processing works and generates a shock wave that propagates outwards radially. At some distance from the works the shock wave has a Mach number of 1.5. Calculate the pressure and velocity just behind the shock wave and comment on its effect.

Solution:

Assume atmospheric conditions: T_1=21°C and p_1 = 101kPa.

Velocity ahead of the shock wave $v_1 = 2\sqrt{kRT} = 2\sqrt{1.4\times287\times294} = 344\text{m/s}$

Now, $(M_a)_2 = \sqrt{\frac{2+(k-1)(M_a)_1^2}{2k(M_a)_1^2-(k-1)}} = \sqrt{\frac{2+\left(0.4\times1.5^2\right)}{\left(2.8\times1.5^2\right)-0.4}} = 0.7$

hence, $T_2 = T_1\times\frac{2+(M_a)_1^2(k-1)}{2+(M_a)_2^2(k-1)} = 294\times\frac{2+\left(1.5^2\times0.4\right)}{2+\left(0.7^2\times0.4\right)} = 388\text{K}$

and $p_2 = p_1\times\frac{\left(1+k(M_a)_1^2\right)}{\left(1+k(M_a)_2^2\right)} = 101\times10^5\,\frac{1+\left(1.4\times1.5^2\right)}{1+\left(1.4\times0.7^2\right)} = 249\times10^5\ \text{Pa}$

Finally, $M_a = \frac{v}{c} = \frac{v}{\sqrt{kRT}}$ so $v_2 = (M_a)_2\sqrt{kRT} = 0.7\sqrt{1.4\times287\times388} = 276\text{m/s}$

So the pressure rises to more than twice atmospheric pressure with winds speeds of 68m/s $(= v_1 - v_2 = 344 - 276)$ or 152 mph. These values will cause extreme damage!