EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D8: Work & Energy

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

Acknowledgements

Many of these examples have arisen through lively discussion in the consortium supported by the NSF grant (#0431756) on "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change" and the input of these colleagues is cheerfully acknowledged as is the support of National Science Foundation.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

PLANE MOTION OF RIGID BODIES

8. Topic**: Work & energy**

Engage:

Take a bag of marbles into class. Empty it onto the bench so that marbles scatter everywhere and fall off the bench. Invite students to collect them and put them on their tables.

Explore:

Ask the students, working in pairs, to identify the types of energy possessed by a marble as it rolls along the bench, falls off the end and hits the floor. Invite some pairs to talk through to the class their understanding of the energy conversions. Remind them about translational kinetic energy ($KE = mv^2/2$) and tell them about the concept of rotational kinetic energy ($KE = I_o\omega^2/2$) about center of mass of a marble.

Explain :

Consider the moment of inertia about the center of mass for a marble of mass 5g & radius 15mm:

$$I = \frac{2}{5}mr^2 = \frac{2}{5}0.005 \times 0.015^2 = 4.5 \times 10^{-7}\,\text{kgm}^2$$

The kinetic energy as it rolls along the bench at 0.5 m/s:

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left(mr^2 + \frac{2}{5}mr^2\right)\omega^2 = \frac{7}{10}mr^2\omega^2$$

where $v = r\omega$ so $\omega = \frac{v}{r} = \frac{0.5}{15 \times 10^{-3}} = 33.3\,\text{rad/s}$

thus, the kinetic energy of a rolling marble is given by:

$$KE = \frac{7}{10}mr^2\omega^2 = \frac{7}{10}0.005 \times 0.015^2 \times 33.3^2 = 8.75 \times 10^{-4}\,\text{J}$$

Elaborate

Now consider a marble starting from a stationary position on the edge of the bench and rolling over the edge.

Applying conservation of energy between two positions: $KE_1 + PE_1 = KE_2 + PE_2$

Position 1: with the center of the marble vertically above the edge (corner) of bench.

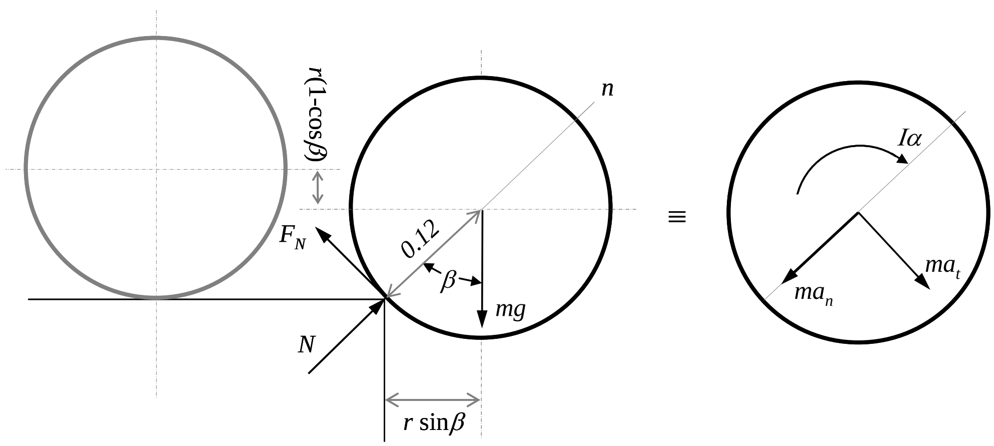
Position 2: when the marble just ceases to make contact with the bench, i.e. it has rolled over the edge to an angle β.

Rearranging the conservation of energy expression, so that the gain in kinetic energy equals loss of potential energy, i.e. $\Delta KE_{21} = \Delta PE_{12}$ and substituting:

$$\frac{7}{10} mr^2 \left(\omega_2^2 - \omega_1^2\right) = mgr(1-\cos\beta)$$

and the marble is initially stationary so $\omega_1 = 0$ and $\omega_2^2 = \dfrac{10g(1-\cos\beta)}{7r}$

Free body diagram at point of losing contact, i.e. position 2:



So resolving forces normal to the contact force at the corner, noting that when contact is lost, $N = F_N = 0$

$$\sum F_n = 0 \text{ so } -mg\cos\beta + ma_n = 0 \text{ and } -mg\cos\beta = -ma_n$$

since $a_n = -\dfrac{v_2^2}{r} = -r\omega_2^2$ then $g\cos\beta = r\omega_2^2$ and $\omega_2^2 = \dfrac{g\cos\beta}{r}$

so equating with expression above from conservation of energy:

$$\frac{g\cos\beta}{r} = \frac{10g(1-\cos\beta)}{7r} \quad \text{and} \quad \cos\beta = \frac{10}{17} \text{ i.e. } \beta = 54\,°$$

The marble loses contact with the bench when it has rolled over by 54 degrees and at this instant it has a velocity:

$$v_2 = r\omega_2 = \sqrt{gr\cos\beta} = \sqrt{9.81 \times 0.015 \times \frac{10}{17}} = 0.294 \text{ m/s}$$

at 54° to the horizontal so that it will land at some distance from the bench, depending on the height of the bench.

Evaluate

Ask students to attempt the following examples:

Example 8.1

A yoyo of diameter 6cm and mass 50g has an axle of diameter 10mm around which a string of length 1.15m is wound. Use the principle of conservation of energy to calculate its angular velocity of the yoyo when it is allowed to fall and unwind under gravity.

Solution

The moment of inertia of the yoyo is: $I = \frac{mR^2}{2} = \frac{0.05 \times 0.03^2}{2} = 2.25 \times 10^{-5}\ \text{kg.m}^2$

Conservation of energy, i.e. from position 1 (start of fall) to position 2 (fully unwound):

$$KE_1 + PE_1 = KE_2 + PE_2$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgh_2$$

or $$mg(h_1 - h_2) = \frac{1}{2}mr^2\omega_2^2 + \frac{1}{2}I\omega_2^2$$

so $$\omega = \sqrt{\frac{2mg(h_1 - h_2)}{mr^2 - I}} = \sqrt{\frac{2 \times 0.05 \times 9.81 \times 1.15}{0.05 \times 0.05^2 - 2.25 \times 10^{-5}}} = 105\ \text{rad/s}$$

which is about 17 revolutions per second.

Example 8.2

A broken down car of mass 1500kg is being pushed up a small incline by a student who can exert a constant force of 600N. After pushing for 5m, the student is joined by a second student who exerts a constant force or 900N and after another 2m a third student joins them and exerts a constant force of 700N but the second student slips and stops pushing only a 1m after the third joins and 2m later the car crests the incline and everyone stops pushing. If the height gained while the car was being pushed was 0.7m, calculate the velocity of the car as it crests the hill.

Solution

Conservation of energy between when the first student starts to push with the car at rest (v_1 = 0) and when everyone stops pushing as the car crests the hill:

$$\Delta KE_{12} + \Delta PE_{12} = \text{Work Done}$$

i.e. $$\frac{1}{2}mv_2^2 + mgh_{12} = \sum \frac{Fd}{2} \quad \text{or} \quad v_2 = \sqrt{\frac{\sum Fd - mgh_{12}}{m}}$$

$$v_2 = \sqrt{\frac{(600 \times 10) + (900 \times 3) + (700 \times 30) - (1500 \times 9.81 \times 0.7)}{1500}} = 2.46\ \text{m/s}$$

The car will crest the hill at 2.5m/s or 5.5 mph).