EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D6: Angular velocity & acceleration

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

Acknowledgements

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

KINEMATICS OF RIGID BODIES

6. Topic: **Angular velocity & acceleration**

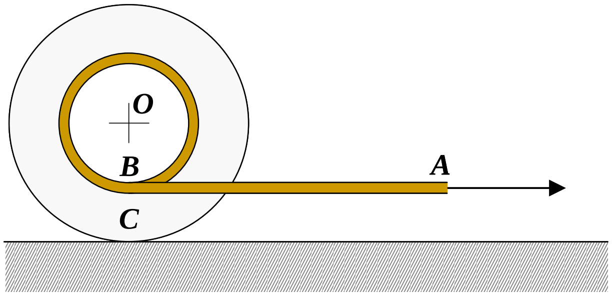
Engage:

Take a yoyo into class and also an empty cable spool with a length of string wound on to it. You should be able to get cable spools from your Department of Electrical Engineering, alternatively ask your campus maintenance staff if their electrical team can collect them for you.

Practice with the yoyo while the class assembles and settles down.

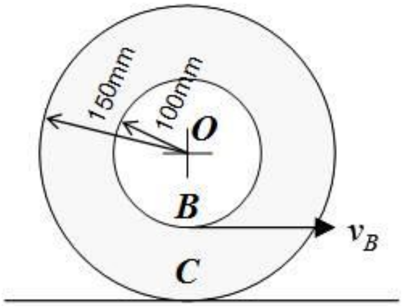
Explore:

Place the spool on the bench with the string partially unwound (orientated as shown in the figure below) and ask the students to predict which way the spool will move when you pull the string and whether the string will be wound or unwound. Collect the options and take a class vote on it before pulling the string carefully so that the spool does not slip. This also works with the yoyo but is less easy to see in a large class.



Explain :

Draw the kinematic diagram:



The spool does not slip so, at contact between spool and table, the velocity is zero, i.e. $v_C = 0$

Velocity is constant all along the unwound string (assume the string is pulled at 150 mm/s) so $v_B = 0.15\,\text{m/s}$ ⇨

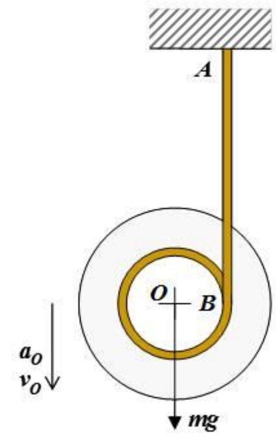
(Angular velocity of spool, $\omega = \dfrac{v_B}{r_{BO}} = \dfrac{0.15}{50\times10^{-3}} = 3$ rad/s ↻

i.e. spool moves in the direction it is being pulled!

Velocity of center of spool, $v_O = r_{OC}\omega = \left(150\times10^{-3}\right)\times 3 = 0.45\,\text{m/s}$ ⇨

So since $v_O \succ v_B$ the string will be wound up at a rate equal to the difference, i.e. 0.3 m/s.

Elaborate



Consider a yoyo falling under gravity when the unwinding of the string causes it to rotate giving it gyroscopic stability, i.e. it will resist forces attempting to move its axis of rotation in the same way as a spinning top (see lesson #11). If the string is attached securely to the axis of the yoyo then, when it is fully unwound, the rotational energy in the yoyo will cause it to start winding back up the string again. The yoyoist will have to pull up on the string slightly in order to replace the energy loss due to friction. In some designs of yoyo, the string is not firmly attached to the axle, in order to allow the yoyo to spin freely or 'sleep' when the string is full unwound, with these designs the yoyoist needs to give a tug to start it winding up again. The tug has the same effect as pulling the string around the spool.

The center of mass of a yoyo will fall at a constant acceleration due to gravity: $a_O = 9.81\text{ m/s}^2$ so at the end of its 1.15m string it will have achieved a velocity given by:

$$v_2^2 - v_1^2 = 2as \qquad \text{and} \qquad (v_O)_2 = \sqrt{2as} = \sqrt{2 \times 9.81 \times 1.15} = 4.75\text{ m/s}$$

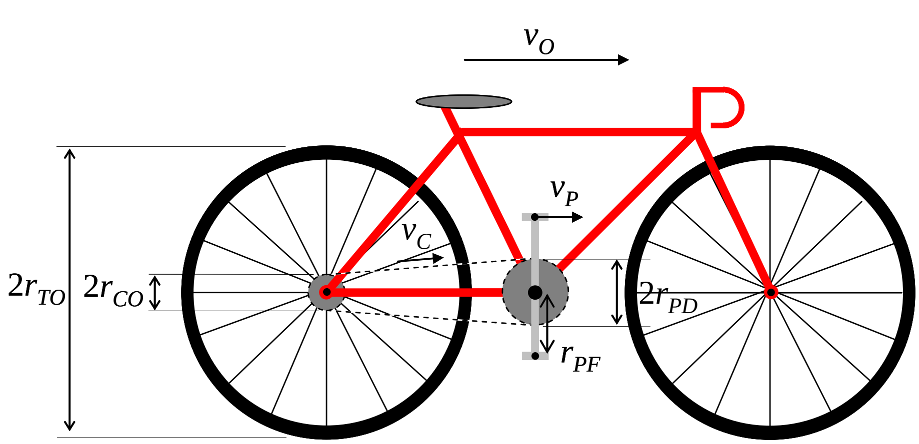
The diameter of the axle is typically 10mm so the rate of rotation is given by,

$$\omega = \frac{v_O}{r_{BO}} = \frac{4.75}{5 \times 10^{-3}} = 950\text{ rad/s} \circlearrowleft$$

This ignores any losses due to friction or air resistance and also ignores the resistance to rotation, i.e. inertia of the yoyo (see example 8.1).

Evaluate

Ask the students to calculate the horizontal velocity of their foot when riding their bicycle at an increasing velocity of 15mph (≡ 6.7m/s) using the middle gear of the range available, at the instant when the pedal is at the top of its rotation.



Solution:

First each student will need to make some measurements on their bike. Typical measurements are given below.

Radius of wheel including tire, r_{TO}: 340 mm

Radius of middle gear at the wheel, r_{CO}: 32 mm

Radius of middle gear at the pedals, r_{PD}: 75 mm

Radius of pedal crank r_{PF}: 175 mm

Consider rotational speed of rear wheel: $\omega_{wheel} = \frac{v_O}{r_{TO}} = \frac{6.7}{0.340} = 19.7\,\text{rad/s}$

Velocity of the chain and the outer radius of the gear at the rear wheel,

$$v_C = r_{CO}\omega_{wheel} = 0.032 \times 19.7 = 0.63\,\text{m/s}$$

The chain will have the same velocity at the pedal gear, so the rotational speed of the pedals will be $\omega_{pedal} = \frac{v_C}{r_{PD}} = \frac{0.63}{0.075} = 8.4$ rad/s

And the velocity of the pedal will be $v_P = r_{PF}\omega_{pedal} = 0.175 \times 8.4 = 1.5\,\text{m/s}$

So your foot would be moving forward with an instantaneous velocity of 1.5m/s (≡ 3.3 mph) which is approximately walking speed!