EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D4: Impulse & momentum

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

KINETICS OF PARTICLES

4. Topic**: Impulse & momentum**

Engage – part I:

Bring a tennis racket and a ball (or two) into class plus a meter rule. Drop a tennis ball on the floor and let it bounce freely. Do this a number of times and ask the class to count the number of bounces before the ball rolls across the floor.

Explore – part I:

Ask the class why the ball does not bounce back to the same height, i.e. why the height of rebound decays. Explain that the ball loses a fraction of its mechanical energy with each impact with the floor.

Since the conservation of energy must apply, ask the class to identify what happens to the lost energy. Explain that it is dissipated in acoustical energy (they can hear the bounce) and heat. Those that have played squash will know that the ball warms up sufficiently during a game to feel the temperature difference.

Note that the sound of the bounce changes with successive bounces, with the impact surface and with the height of the initial drop. Use the meter rule to drop it from different heights in order to illustrate this effect.

Explain – part I:

A ball of mass, m on released from a height, h has a potential energy, $PE = mgh$; after hitting the floor it bounces to height, d where it has a potential energy, $PE = mgd$. The coefficient of restitution, $e = \sqrt{(d/h)}$ and is equal to the fraction of mechanical energy lost.

This fraction is the same for successive bounces, i.e.

$$e^2 = d_1/h = d_2/d_1 = d_3/d_2 = d_4/d_3$$

where d_1, d_2, d_3, d_4 are the heights of successive bounces.

The coefficient of restitution of a tennis ball bouncing on a concrete floor is about 0.71 so if the ball is dropped from 1m the height of successive bounces will be

$d_1 = e^2h = 0.71^2 \times 1 = 0.5$m and $d_2 = 0.25$m, $d_3 = 0.13$m, $d_4 = 0.07$m…

Engage – part II:

Place a second tennis ball on a bench and roll the first one into it with sufficient momentum to cause them both to move after the impact.

Explore – part II:

Explain how conservation of energy causes kinetic energy from the moving ball, before impact to be converted to strain energy of deformation for both balls whilst they are in contact and then restituted as kinetic energy in both balls after impact. The coefficient of restitution can also be defined in terms of velocities, so for two balls (particles), A and B:

$$e = \frac{(v_B)_{after} - (v_A)_{after}}{(v_A)_{before} - (v_B)_{before}}$$

Elaborate

In a tennis machine (for an example goto www.youtube.com and type in tennis machine[4]) tennis ball A rolls down a chute from a vertical height of 0.25m and impacts tennis ball B at the bottom of the chute, we can calculate the velocity at the instant before impact by conservation of energy. The kinetic energy of the ball, A at the bottom of the chute equals its potential energy at the top, i.e. $\frac{1}{2} m_A (v_A)^2_{before} = m_A gh$ so

$$(v_A)_{before} = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.25} = 2.2 \text{ m/s}$$

Now considering the coefficient of restitution in terms of velocities, assuming ball B is stationary before the impact:

$$e = \frac{(v_B)_{after} - (v_A)_{after}}{(v_A)_{before} - (v_B)_{before}} = \frac{(v_B)_{after} - (v_A)_{after}}{2.2 - 0} \text{ so } (v_B)_{after} = 1.6 + (v_A)_{after}$$

Now applying the principle of conservation of momentum:

$$m_A (v_A)_{before} + m_B (v_B)_{before} = m_A (v_A)_{after} + m_B (v_B)_{after} \text{ and } \quad m_A = m_B$$

so $\quad (v_A)_{before} + (v_B)_{before} = (v_A)_{after} + (v_B)_{after}$

and $\quad 2.2 + 0 = (v_A)_{after} + (1.6 + (v_A)_{after}) \quad$ or $\quad (v_A)_{after} = \frac{2.2 - 1.6}{2} = 0.3 \text{ m/s}$

and $\quad (v_B)_{after} = 2.22 + (v_A)_{after} = 1.6 + 0.3 = 1.9 \text{ m/s}$

In other words the stationary ball moves off at 1.9m/s and the impacting ball at 0.3m/s.

Evaluate

Ask students to attempt the following examples:

Example 4.1

A 4 year-old boy weighing 15kg is sitting on the bottom edge of a waterslide (with his legs dangling in the swimming pool) when his 7 year-old sister weighing 22kg comes down the slide

[4] http://www.youtube.com/watch?v=EWNq0t6ap-8

from a height of 3m. Assuming the end of the slide is horizontal and neglecting friction, calculate the velocity of the boy after impact if (a) his sister grabs him and they move off together and (b) if they separate after contact with a coefficient of restitution of 0.8.

Solution:

(a) For the girl (subscript 'G'), the kinetic energy before impact is equal to her potential energy at top of slide:

So $\frac{1}{2} m_G (v_G)^2_{before} = m_G gh$

and $(v_G)_{before} = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 3} = 7.7$ m/s

So in case (a) applying conservation of momentum using subscript 'B' for the boy:

$$m_B (v_B)_{before} + m_G (v_G)_{before} = m_B (v_B)_{after} + m_G (v_G)_{after}$$

with $(v_B)_{after} = (v_G)_{after} = v_{after}$

$$(15 \times 0) + (22 \times 7.7) = (15 + 22) \times v_{after}$$

and $v_{after} = \dfrac{22 \times 7.7}{15 + 22} = 4.56$ m/s

i.e. they will shoot off the slide together at a velocity of 4.6m/s (10 mph) horizontally.

(b) for a coefficient of restitution of e = 0.8

$$e = \frac{(v_B)_{after} - (v_G)_{after}}{(v_G)_{before} - (v_B)_{before}} = \frac{(v_B)_{after} - (v_G)_{after}}{7.67 - 0}$$

and $(v_B)_{after} = 7.67e + (v_G)_{after}$

now applying conservation of momentum:

$$m_B (v_B)_{before} + m_G (v_G)_{before} = m_B (v_B)_{after} + m_G (v_G)_{after}$$

thus $(15 \times 0) + (22 \times 7.67) = 15 \times (7.7e + (v_G)_{after}) + 22 (v_G)_{after}$

and $(v_G)_{after} = \dfrac{(22 \times 7.67) - (15 \times 7.7 \times 0.8)}{(15 + 22)} = 2.1$ m/s

so $(v_B)_{after} = 7.7e + (v_G)_{after} = 6.2 + 2.1 = 8.3$ m/s

i.e. the boy slides off the slide at 8 m/s (18 mph) and the girl follows at 2m/s (4 mph).

Example 4.2

Ask students to identify other events with which they are familiar that can be analyzed in this way and then to select one and perform a similar analysis to the one above