EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D2: Force & acceleration

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

Acknowledgements

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

KINETICS OF PARTICLES

2. <u>Topic</u>: **Force and acceleration**


Photo by John Gwillim/The State News

Engage:

Take an ice hockey stick and puck into class and push the puck around on the floor. If you know nothing about ice hockey shots then try typing 'ice hockey snapshot' in www.youtube.com for some short coaching videos[4]. Alternatively use a dustpan and broom with a jar lid (e.g. a jam jar lid). Sweep the jar lid into the dustpan.

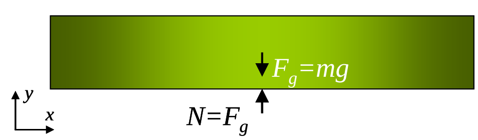
Explore:

Discuss the meaning of 'kinetics' which involves the analysis of forces causing motion. Remind the class about how consideration of the drag force gave a more realistic answer for the velocity of the raindrops in the previous lesson. Discuss the forces acting on the puck (jar lid) when it is pushed across the floor: driving force from the stick (broom); friction; drag (probably negligible), gravity and the reaction from the floor.

Explain:

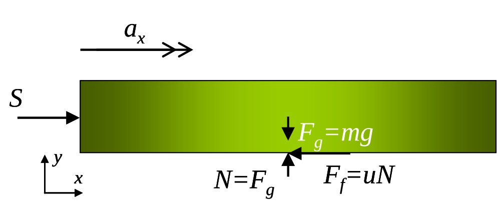
Sketch the free body diagrams below explaining Newton's laws of motion as the situation becomes progressively more complicated:

<u>Puck resting on ice (just before being push)</u>



Newton's Third Law: *for every action, there is an equal and opposite reaction*. So the mass of the puck acts on the floor and there is an equal and opposite reaction from the floor, *N*.

<u>Puck being pushed across the floor by stick</u>



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[4] http://www.youtube.com/watch?v=TYEE7tZhRtk

Newton's Second Law: *the acceleration of an object is proportional to the force acting on it and inversely proportional to its mass.* So when a force, S is applied with the stick, the puck will accelerate in the direction of this force, such that:

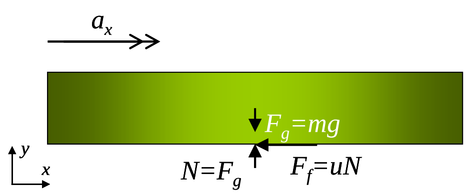
$$\sum F_x = ma_x$$

There will be also a frictional force resisting motion at the interface between the floor and the bottom of the puck:

$$F_f = \mu N$$

where μ is the coefficient of friction.

<u>Puck sliding across the ice on its own</u>



Newton's First Law: *a body continues to maintain its state of rest or of uniform motion unless acted upon by an external force.* This also applied to the first diagram in which the puck is in a state of rest. In this case if the floor is highly polished or the puck is travelling on ice then the coefficient of friction could be assumed to be almost zero and the acceleration will be approximately zero; otherwise the puck will suffer a deceleration due to the friction force from the floor.

Elaborate

An NHL ice hockey puck has a maximum mass of 170g. So if a player applied a force of 35N and the coefficient of friction between the ice and the puck (vulcanized rubber) is approximately 0.15 then applying Newton's Second Law:

$$\sum F_x = ma_x \text{ and } a_x = \frac{\sum F_x}{m} = \frac{S - \mu N}{m} = \frac{35 - (0.15 \times 0.170 \times 9.81)}{0.170} = 204 \text{ m/s}^2$$

and if the player pushes it for 1.5m then the velocity as the puck leaves the stick will be:

$$v_x^2 - v_{x=0}^2 = 2as\text{, i.e. } v_x = \sqrt{2as} = \sqrt{2 \times 204 \times 1.5} = 24.8 \text{ m/s } (\equiv 55 \text{ mph})$$

Alternatively, for a jar lid of mass 15g sliding on a wooden floor (μ = 0.2) assuming a force of only 1N is applied with the broom then:

$$a_x = \frac{S - \mu N}{m} = \frac{1 - (0.2 \times 0.015 \times 9.81)}{0.015} = 65 \text{ m/s}^2$$

And if it is pushed by the broom for 0.05m (≈ 2 inches) then its velocity will be:

$$v_x = \sqrt{2as} = \sqrt{2 \times 65 \times 0.05} = 2.54 \text{ m/s } (\equiv 5.7 \text{ mph})$$

Evaluate

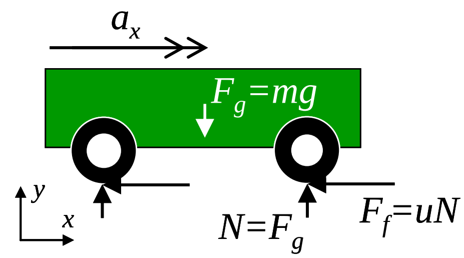
Ask students to attempt the following examples:

Example 2.1

Sometimes during half-time at a hockey game, a car is driven into the ice-ring (perhaps to show it off as a lottery prize). If the driver applies the brakes so that they lock, calculate the maximum speed for which the skid will be no more than 2m if the car has a mass of 2000kg. Compare this with a skid on a wet road (μ=0.45)

Solution:

Free-body diagram:



applying Newton's Second Law for car on ice:

$$\sum F_x = ma_x \text{ and } a_x = \frac{F_f}{m} = \frac{\mu N}{m} = \frac{(0.15 \times 2000 \times 9.81)}{2000} = 1.47 \text{ m/s}^2$$

Using kinematics: $v_x{}^2 - v_{x=0}^2 = 2as$, i.e. $v_{x=0} = \sqrt{2as} = \sqrt{2 \times 1.47 \times 2} = 2.4$m/s (≡5.4 mph)

Again applying Newton's Second Law for car on wet concrete:

$$\sum F_x = ma_x \text{ and } a_x = \frac{\mu N}{m} = \frac{(0.45 \times 2000 \times 9.81)}{2000} = 4.41 \text{ m/s}^2$$

Using kinematics: $v_x{}^2 - v_{x=0}^2 = 2as$, i.e. $v_{x=0} = \sqrt{2as} = \sqrt{2 \times 4.41 \times 2} = 4.2$ m/s (≡9.4 mph)

This is why cars are fitted with Antilock Brake Systems (ABS) !

Example 2.2

Ask the students to continue exploring the lesson example by calculating the maximum distance from the goal-keeper that the shot can be taken in order to beat the reactions of the goal-keeper, assuming a typical visual reflex time is 180 milliseconds.

Solution:

For the puck moving across the ice,

$$\sum F_x = ma_x \text{ and } a_x = \frac{F_f}{m} = \frac{\mu N}{m} = \frac{(0.15 \times 0.170 \times 9.81)}{0.170} = 1.47 \text{ m/s}^2$$

The distance that the puck will travel in 180 milliseconds is:

$$s = v_x\, t + \tfrac{1}{2} a_x t^2 = (24.8 \times 0.180) + (0.5 \times (-1.47) \times 0.18^2) = 4.4 \text{ m}$$

So the shot should be taken within 4m of the goal-keeper.