EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D1: Rectilinear & curvilinear motion

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

KINEMATICS OF PARTICLES

1. Topic: **Rectilinear & Curvilinear motion**

Engage:

Stop by the office recycling box on your way to class and pick-up enough sheets of paper to give one to each member of the class and have some left over. Pass them around the around the class and invite the students to write on their sheet three reasons why they chose to study engineering; then ask them to throw it to a member of the class on the other side of the room.

Most of the students will crunch the sheet up into ball to throw it, and a few might make a paper airplane. If no one else does, you should demonstrate what happens if you try to throw it as a sheet of paper.

Explore:

Discuss what is meant by the term 'kinematics', i.e. cases in which only the geometric aspects of motion are considered and that a particle has mass but negligible size and shape (no air resistance). For the sheets of paper to be considered particles then their dimensions must have no influence on the analysis of their motion, i.e. their motion is characterized by the motion of their center of mass and any rotation of the body can be neglected. So, the flat sheets don't fly well due to the effect of air resistance on their shape but the tight balls behave as particles and go where you aim them. The trajectory of the paper airplanes is strongly dependent on their shape.

Explain:

In a heavy rainstorm large droplets fall from low Nimbostratus cloud at 600 ft (183m). Let's assume that raindrops are approximately spherical and of diameter, $d \approx 3$mm, so given the density of water is 1000 kg/m^3, their mass (m) will be:

$$m = \rho_{H_2O} V = \rho_{H_2O} \frac{4\pi\ r^3}{3} = 1000 \times \frac{4\pi \times 0.0015^3}{3} = 1.413 \times 10^{-5}\,\text{kg}\ (\equiv 0.014\text{g})$$

Assuming a constant acceleration, g = 9.81m/s^2, and that the time at the start of the fall is $t = 0$ when velocity, $v_y = v_{y=0} = 0$, and displacement, $s_y = s_{y=0} = 0$ then when the raindrops fall 183m we find using:

$$v_{y=183}^2 - v_{y=0}^2 = 2a_y(s_{y=183} - s_{y=0})$$

that $$v_{y=183} = \sqrt{2gs_{y=183}} = \sqrt{2 \times 9.81 \times 183} = 59\,\text{m/s}$$

This is 212 km/hr or 132 mph and is probably unrealistic.

In practice we need to include an analysis of the forces causing motion, i.e. **kinetics** and calculate a terminal velocity, v_t which is approached when the drag force, F_d due to air resistance[♠] equals the gravitational force, F_g (= mg), i.e. $F_d = F_g$

where $F_d = \frac{1}{2}\rho_{air} v^2 A C_d$

and the density of air, $\rho_{air} \approx 1.2$ kg/m^3, A is the frontal area and C_d is the drag coefficient (≈ 0.4 for a rough sphere).

Thus $mg = \frac{1}{2}\rho_{air} v^2 A C_d$ and $v_{yt} = \sqrt{\frac{2mg}{\rho_{air} A C_d}} = \sqrt{\frac{2\times(1.413\times10^{-5})\times 9.81}{1.2\times(\pi/4)(3\times10^{-3})^2\times 0.4}}$ = 9 m/s

Elaborate

The horizontal dispersion of the rain droplets can be estimated by considering the horizontal component of the motion after calculating, in the vertical direction, the time from release to impact with the ground, i.e. $s = vt + \frac{1}{2}at^2$ but at terminal velocity the acceleration is zero,

so $s_y = v_{yt}\, t$ and $t = \frac{s_y}{v_{yt}} = \frac{183}{9} = 20.3\,\text{s}$

Plenty of time to dodge it if there was only one! Considering the horizontal direction with a constant wind speed of 22m/s (≈ 50 mph – a force 10 gale according the Beaufort scale) then

$$s_x = v_{x=0}t + \tfrac{1}{2}a_x t^2 = 22\times 20.3 = 447\,\text{m}$$

i.e. it travels further horizontally than it falls vertically which is what we often see in a violent storm. If the wind drops to force 3, more likely with Nimbostratus, with a wind speed of 10mph (≈ 4.5 m/s) then they fall much closer to the vertical, i.e.

$$s_x = v_{x=0}t + \tfrac{1}{2}a_x t^2 = 4.5\times 20.3 = 91.4\,\text{m}$$

Evaluate

Invite students to attempt the following examples:

Example 1.1

An individual, who is 1.7m (≈ 5ft 6in) tall, tilts her head back to sneeze so that particles leave her nostrils horizontally with a velocity of 40m/s (≡ 90mph), how far away from the individual will the particles land?

Solution:

Time to fall vertically: $s_y = v_{y=0}t + \frac{1}{2}a_y t^2$ so $t = \sqrt{\frac{2s_y}{a_y}} = \sqrt{\frac{2\times1.7}{9.81}} = 0.589\,\text{s}$ given $v_{y=0} = 0$

Distance travelled horizontally: $s = v_{x=0}t + \frac{1}{2}a_x t^2 = (40\times 0.589) + 0 = 23.5\,\text{m}$

♠ It is assumed that the upwards force of buoyancy is negligible.

Example 1.2

While riding your bike across a pedestrian bridge at about 50km/hr (≈30mph) your iPod drops out of your pocket and, missing the bridge, falls to the ground 6m below. Estimate the velocity at which your iPod will hit the ground below the bridge and the horizontal distance that it will travel before impact. Comment on the factors influencing whether your iPod will survive the fall.

Solution:

Time to fall vertically: $s_y = v_{y=0}t + \frac{1}{2}a_y t^2$ so $t = \sqrt{\frac{2s_y}{a_y}} = \sqrt{\frac{2\times 6}{9.81}} = 1.11\,\mathrm{s}$ given $v_{y=0} = 0$

Distance travelled horizontally:

$$s = v_{x=0}t + \frac{1}{2}a_x t^2 = \left(\frac{50\times 1000}{60\times 60}\times 1.11\right) + 0 = 13.89\times 1.11 = 15.4\ \mathrm{m}$$

Vertical component of velocity on impact:

$$v_y - v_{y=0} = a_y t \ \text{ so } \ v_y = a_y t = (9.81\times 1.11) = 10.9\ \mathrm{m/s}$$

Magnitude of velocity on impact, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{13.9^2 + 10.8^2} = 17.7\ \mathrm{m/s}$

Direction of impact, $\alpha = \tan^{-1}\frac{10.9}{13.9} = 38°$

So your iPod will hit the ground at 18m/s (40mph) at an angle of 38° to the ground.