# EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

## S8: Beam bending

*This is an extract from 'Real Life Examples in Mechanics of Solids: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".*

## INTRODUCTION

(from *'Real Life Examples in Mechanics of Solids: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore course in mechanics of solids, increase the accessibility of the principles and raise the appeal of the subject to students from a diverse background[1]. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. These are not original and were developed by the Biological Sciences Curriculum Study[2] in the 1980s from work by Atkin and Karplus[3] in 1962. Today they are considered to form part of the constructivist learning theory and a number of websites provide easy to follow explanations of them[4].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

**Acknowledgements**

Many of these examples have arisen through lively discussion in the consortium supported by the NSF grant (#0431756) on "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change" and the input of these colleagues is cheerfully acknowledged as is the support of NSF. The influence of the editor's mentors and peers at the University of Sheffield is substantial and is gratefully acknowledged since many of the ideas for these examples originate from tutorial questions developed and used in the Department of Mechanical Engineering in Sheffield over many years.

Eann A. Patterson
*A.A. Griffith Chair of Structural Materials and Mechanics*
*School of Engineering, University of Liverpool, Liverpool, UK*
*& Royal Society Wolfson Research Merit Award Recipient*

[1] Patterson, E.A., Campbell, P.B., Busch-Vishniac, I., Guillaume, D.W., 2011, The effect of context on student engagement in engineering, *European J. Engng Education*, 36(3):211-224.
[2] http://www.bscs.org/library/BSCS_5E_Instructional_Approach_July_06.pdf
[3] Atkin, J. M. and Karplus, R. (1962). Discovery of invention? *Science Teacher* 29(5): 45.
[4] e.g. http://www.science.org.au/primaryconnections/constructivist.htm

**BEAM BENDING**

8. Principle: **Bending moments and shear force diagrams**

**Engage:**

Ride a skateboard into class.

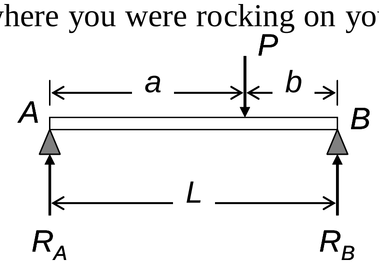
**Explore:**

Discuss the shear forces and bending moments set-up in the skateboard when you stand on it sideways balanced on your heels, i.e. approximating a point load. When you stand of the board more normally, how do the shear forces and bending moments change? Discuss where you need to stand to induce a zero bending moment.

You might want to ask students to work in pairs to draw schematics of these loading schemes.

**Explain:**

Plot the shear force and bending moment diagrams for the case where you were rocking on your heels

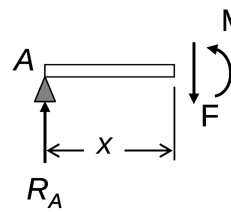



Considering the complete beam

Resolve vertically: $R_A + R_B = P$

Moments about A: $Pa - R_B L = 0$

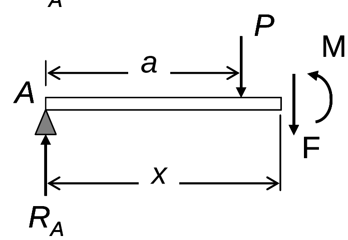
Thus: $R_A = \frac{Pb}{L} \quad R_B = \frac{Pa}{L}$




Considering the cut section (0<*x*<*a*)

Resolve vertically: $F = R_A = \frac{Pb}{L}$

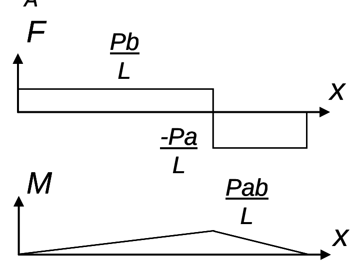
Moments: $M = R_A x = \frac{Pbx}{L}$




Considering the cut section (a<*x*<*l*)

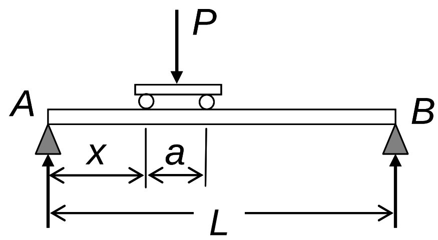
Resolve vertically: $F = R_A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$

Moments: $M = R_A x - P(x-a) = \frac{Pa}{L}(L-x)$



**Elaborate**

When a skateboarder crosses a plank we can determine the position at which the bending moment is a maximum. The situation can be idealized as shown below:



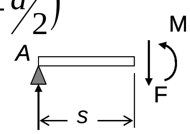

The shear force and bending moment diagrams can be plotted as previously considering small sections of beam, i.e.

Taking moments about B gives:

$$R_A L - \frac{P}{2}(L - x - a) - \frac{P}{2}(L - x) = 0$$

Thus, $R_A = \frac{P}{L}\left(L - x - \frac{a}{2}\right)$

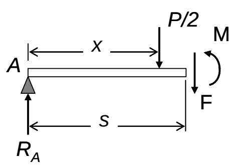
For $(s < x)$




Resolving vertically: $F = -R_A$

Taking moments: $M = -R_A s$

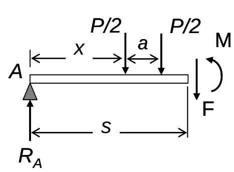
For $(x < s < (a + x))$




Resolving vertically: $F = \frac{P}{2} - R_A$

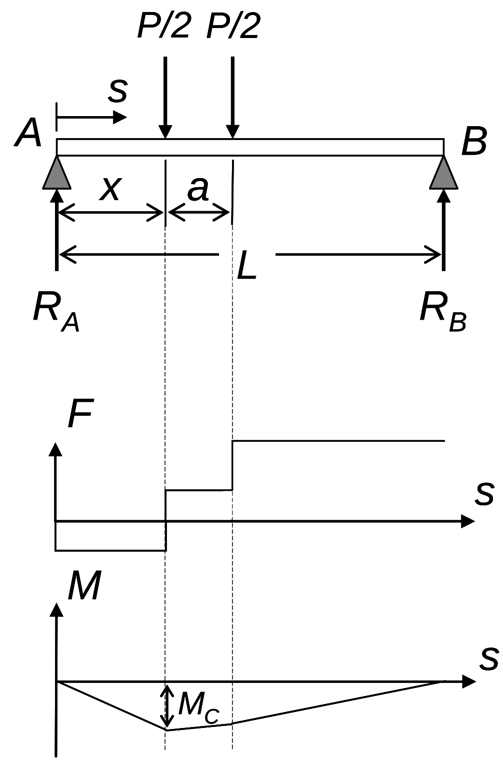
Taking moments: $M = \frac{P}{2}(s - x) - R_A s$

For $(s > (a + x))$




Resolving vertically: $F = P - R_A$

Taking moments: $M = P\left(s - x - \frac{a}{2}\right) - R_A s$




*N.B. These diagrams have been plotted assuming that $(x + a/2) > L/2$, if this were not the case then the diagrams would look slight different. The symmetry of the situation allows only this case to be considered. There are two places where $M_c$ occurs that are symmetric about the mid-point of the beam*

For maximum $M_C$:

$$\frac{\partial M_C}{\partial x} = 0 \text{ and } M_C = R_A x = \frac{Px}{2L}(2L - 2x - a)$$

So, $$\frac{\partial M_C}{\partial x} = 2L - 4x - a = 0 \text{ and } x = \frac{L}{2} - \frac{a}{4}$$

Thus, $$\hat{M}_C = \frac{P}{2L}\left(2L - 2\left(\frac{L}{2} - \frac{a}{4}\right) - a\right)\left(\frac{L}{2} - \frac{a}{4}\right) = \frac{P}{16L}(2L - a)^2$$

So for a 1.8m plank and typical skate board ($a$=65cm) carrying a 65kg person,

$$\hat{M}_C = \frac{65 \times 9.81}{16 \times 3}(3.6 - 0.65)^2 = 116\,\text{Nm}$$

If the plank is 13cm wide and 1.8cm thick, then the maximum bending stress is

$$\sigma = \frac{\hat{M}_C y}{I} = \frac{\hat{M}_C (h/2)}{(bh^3/12)} = \frac{6 \times 116}{0.13 \times 0.018^2} = 16.5\,\text{MPa}$$

This compares to compressive ultimate strengths for common woods in the range 35 to 55 MPa parallel to the grain and 4 to 10MPa perpendicular to the grain.

**Evaluate**

*Example 8.1*

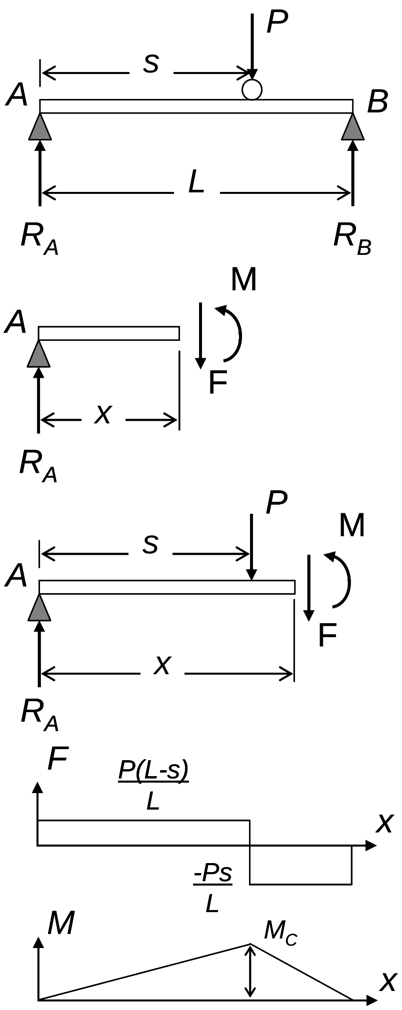
Ask students to repeat the analysis above but for unicyclist crossing the plank. See next page for solution.

*Example 8.2*

Ask students to look for two other examples in their everyday life and explain how the above principles apply to each example.

*Example 8.1*

Solution:




Considering the complete beam

Resolve vertically: $R_A + R_B = P$

Moments about A: $Ps - R_B L = 0$

Thus: $R_A = \frac{P(L-s)}{L}$ $R_B = \frac{Ps}{L}$

Considering the cut section (0<$x$<$s$)

Resolve vertically

: $F = R_A = \frac{P(L-s)}{L}$

Moments:

$$M = R_A x = \frac{P(L-s)x}{L}$$

Considering the cut section (s<$x$<$L$)

Resolve vertically:

$$F = R_A - P = \frac{P(L-s)}{L} - P = -\frac{Ps}{L}$$

Moments:

$$M = R_A x - P(x-s) = \frac{Ps}{L}(L-x)$$

For maximum bending moment:

$$\frac{\partial M_C}{\partial s} = 0 \text{ and } M_C = R_A x = \frac{P(L-s)s}{L} = \frac{P}{L}\left(Ls - s^2\right)$$

So, $$\frac{\partial M_C}{\partial s} = L - 2s = 0 \text{ and } s = \frac{L}{2}$$

Thus, $$\hat{M}_C = \frac{P}{L}\left(\frac{L^2}{2} - \frac{L^2}{4}\right) = \frac{PL}{4} = \frac{65\times 9.81\times 1.8}{4} = 287\,\text{Nm}$$

$$\sigma_{\max} = \frac{\hat{M}_C y}{I} = \frac{\hat{M}_C (h/2)}{\left(bh^3/12\right)} = \frac{6\times 287}{0.13\times 0.018^2} = 41\text{MPa}$$

Maximum stress of 41MPa induced when unicyclist at the middle. The position could have been deduced without analysis.