EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

S1: Elementary stress systems

This is an extract from 'Real Life Examples in Mechanics of Solids: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Mechanics of Solids: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore course in mechanics of solids, increase the accessibility of the principles and raise the appeal of the subject to students from a diverse background[1]. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. These are not original and were developed by the Biological Sciences Curriculum Study[2] in the 1980s from work by Atkin and Karplus[3] in 1962. Today they are considered to form part of the constructivist learning theory and a number of websites provide easy to follow explanations of them[4].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

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[1] Patterson, E.A., Campbell, P.B., Busch-Vishniac, I., Guillaume, D.W., 2011, The effect of context on student engagement in engineering, *European J. Engng Education*, 36(3):211-224.
[2] http://www.bscs.org/library/BSCS_5E_Instructional_Approach_July_06.pdf
[3] Atkin, J. M. and Karplus, R. (1962). Discovery of invention? *Science Teacher* 29(5): 45.
[4] e.g. http://www.science.org.au/primaryconnections/constructivist.htm

ELEMENTARY STRESS SYSTEMS

1. Principle: **Stress and strain in uni-axial solid and hollow bars**

Engage:

Take your iPod into class and dangle it by the earphone cable. Cut open the cable on an old set of earphones to expose cable and insulation.

Explore:

Pass around class lengths of copper wire and lengths of empty hollow insulation and invite students to stretch them. Discuss relative extensions and stiffness. Someone will probably snap one so talk about ultimate tensile stress. Be sure have to enough lengths that every student has at least one to play with while you are talking.

Explain:

Work through the example below:

An iPod, with a mass of 30 grams is dangled from its earplug cable.

a) Assuming that the copper wire of diameter 0.40mm inside the cable carries the entire load, evaluate the stress in the wire due to the weight of the iPod.

b) If the wire in (a) is 1.50m long, by how much will it stretch?

c) Assuming that the plastic insulation, which fits snugly over the wire and has an external diameter of 1mm, carries the entire load, evaluate the stress in the insulation due to the weight of iPoD.

d) If the insulation is made from uPVC calculate the extension of the insulation in the circumstances described in (c).

Solution:

a) Stress, $\sigma = \frac{F}{A} = \frac{mg}{\left(\pi d^2/4\right)} = \frac{\left(30\times10^{-3}\right)(9.81)}{\pi\times\left(0.40\times10^{-3}\right)^2/4} = 2.34\times10^{6}\text{N/m}^2$

where F, is force applied, A is cross-section area, m is mass, g is gravitational acceleration, and d is the diameter of the cross-section.

b) Extension, $\delta = \varepsilon L = \frac{\sigma L}{E} = \frac{FL}{AE} = \frac{mgL}{E\pi d^2/4} = \frac{\left(30\times10^{-3}\right)(9.81)(1.50)}{\left(110\times10^{9}\right)(\pi)\left(0.40\times10^{-3}\right)^2/4} = 31.9\times10^{-6}\text{m}$

where L is the length of the wire and E is the Young's modulus of copper obtained from a data book .

c) Stress, $\sigma = \dfrac{F}{A} = \dfrac{mg}{\left(\pi\left(d_o^2 - d_i^2\right)/4\right)} = \dfrac{\left(30\times10^{-3}\right)(9.81)}{\pi\left(0.001^2 - 0.004^2\right)/4} = 0.29\text{N/m}^2$

where d_o and d_i are the outer and inner diameters of the insulation.

d) Extension, $\delta = \dfrac{FL}{AE} = \dfrac{mgL}{E\pi\left(d_o^2 - d_i^2\right)/4} = \dfrac{\left(30\times10^{-3}\right)(9.81)(1.5)}{\left(2\times10^9\right)(\pi)\left(1^2 - 0.4^2\right)\times10^{-6}/4}$

$=3.35\times10^{-4}\text{m}$

Elaborate:

In practice the load will be borne by the wire and insulation together, discuss how this will influence the extension of both of them. The copper and plastic are bound together and must extend by the same amount, i.e. $\delta_{wire} = \delta_{insulation}$. Consequently the wire will extend less and the insulation extend more; causing more tension in the insulation than calculated and less tension in the wire.

Evaluate:

Invite the students to undertake the following examples

Example 1.1

At the center of a suspension bridge, the 20mm diameter suspension cables supporting the deck are 30m long. Calculate the extension of the steel cables when a 44tonne truck passes along the deck if this load is shared between twelve cables on each side of the deck.

Solution:

Asked for extension due to truck and hence can ignore the loading due to the weight of the deck:

Change in extension, $\delta = \dfrac{FL}{nAE} = \dfrac{mgL}{nE\pi d^2/4} = \dfrac{\left(44\times10^3\right)(9.81)(30)}{24\times\left(21\times10^{10}\right)(\pi)\left(20\times10^{-3}\right)^2/4} = 0.0082\text{m}$

where F is the applied load, L the length of the cable, n number of cables carrying load, A is a cable cross-section area, E is the Young's modulus of steel from a databook, and d is the cable diameter.

Example 1.2

a) Estimate the stress in your femur when standing still and upright with your weight distributed evenly on both feet.

b) Repeat the exercise in (a) for an adult African elephant and for an adult mouse. Only rough estimates of the bone dimensions and mass are necessary.

c) Assuming that the strength of bone in humans, elephants and mice is approximately equal, discuss the relative susceptibility to fractures.

Solution

a) Stress, $\sigma = \frac{F}{A} = \frac{mg}{n\left(\pi d^2/4\right)} = \frac{(80)(9.81)}{2\pi \times \left(25\times 10^{-3}\right)^2/4} = 800{,}000\ \text{N/m}^2$

where F, is force applied, A is cross-section area of the bone, m is mass, g is gravitational acceleration, d is the diameter respectively of the cross-section and n is the number of legs.

b) Repeat for an elephant with for example: n=4, m=6 tonnes and d= 200mm, giving σ= 360,000 N/m^2; and for a mouse with for example n=4, m=25 grams and d=1mm giving σ= 78,600 N/m^2

c) The stress is lower in the legs of the mouse so that they are much less likely to break their legs.

Example 1.3

On a cello the 0.68m long steel strings are tuned by winding one end around a peg or fret. For peg diameter of 15mm, calculate how many turns will be necessary to achieve a tension in the 1.36mm diameter string of 84 N (approx middle G)

(see http://gamutstrings.com/tensions/cloten.htm for more information of cello strings).

Solution

Extension, $\delta = \varepsilon L = \frac{\sigma L}{E} = \frac{FL}{AE} \frac{84\times 0.68}{\left(\pi \times \left(1.36\times 10^{-3}\right)^2/4\right)\times\left(210\times 10^{9}\right)} = 0.00019\text{m}$

And this wrapped is around the circumference of the peg,

$$n = \frac{0.00019}{\pi d} = \frac{0.00019}{\pi\left(15\times 10^{-3}\right)} = 0.003 \text{ times or about 1.4 degrees.}$$

The peg needs to be turned 1.4 degrees.

Example 1.4

Ask students to look for two other examples in their everyday life and explain how the above principles apply to each example.